WORLD CIST 15
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University of Minho
School of Engineering

Concurrency Detection on Finish-to-Start Activity Precedence Networks

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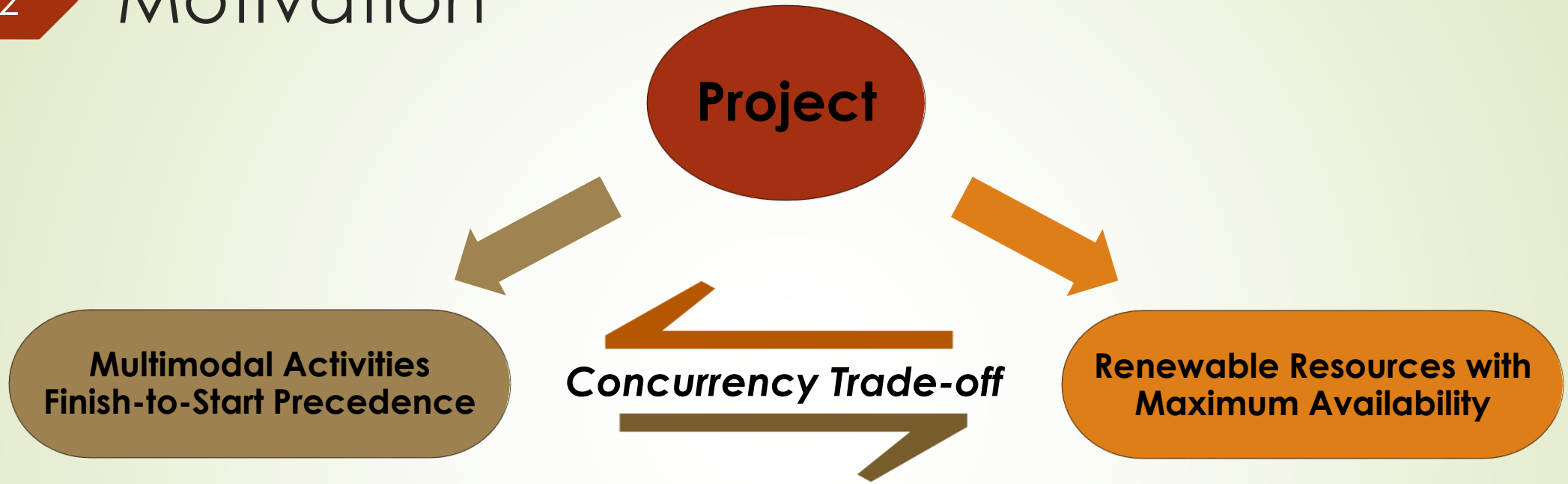
Motivation



- Allows concurrency
- Resource requirement

- Fixed availability (at all times)
- Fully released after activity completion

To assess the impact of the availability constraint

Concurrency Detection Method

Collection

- Precedence relation

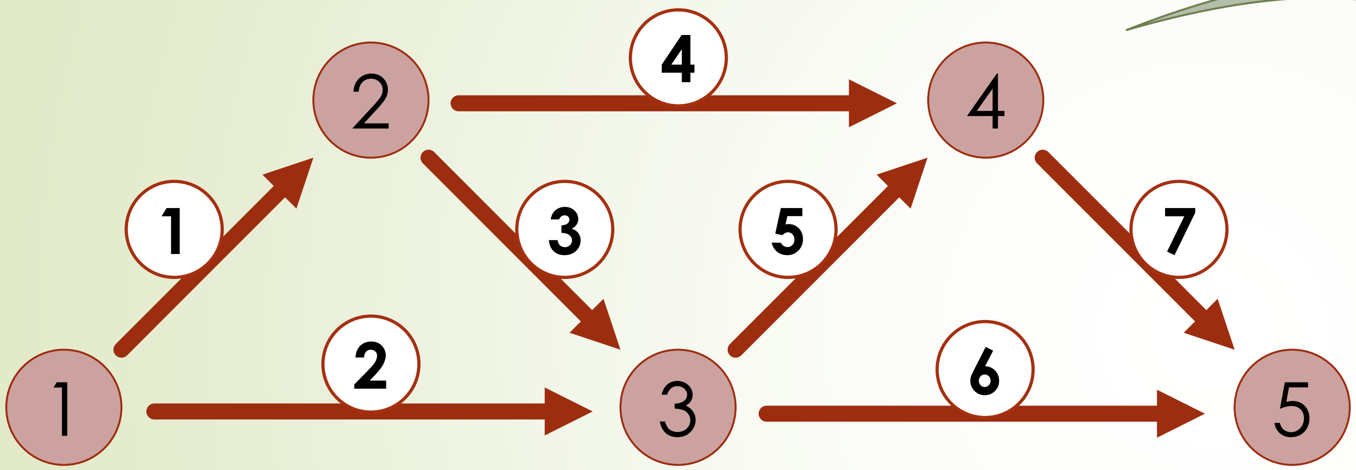
Transformation

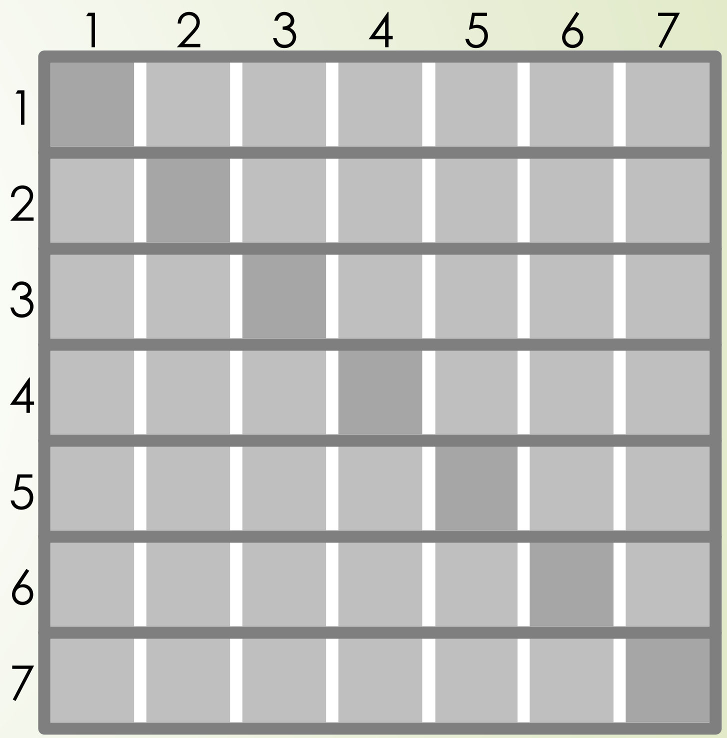
- Parallel relation

Identification

- Sets of project activities that can execute concurrently with each other

Detection Method: Collection





Finish-to-Start Precedence (matrix form)

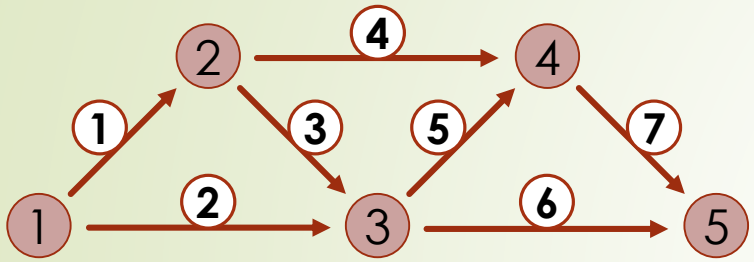
Transitive Precedence (matrix form)

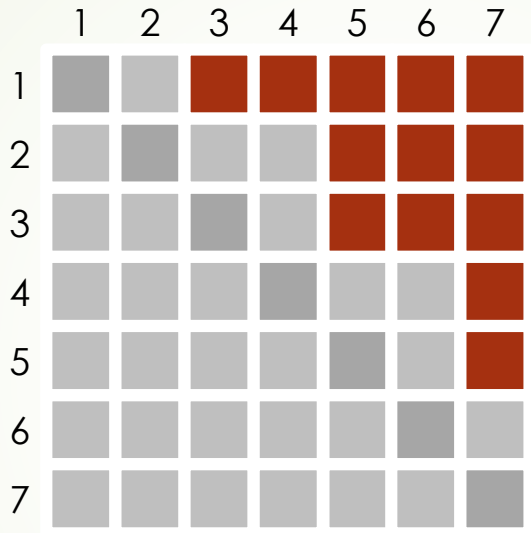
$row \times column =$ ■ reads:
"row" must precede "$column$"

Finish-to-Start Activity Precedence Relation
usually specified through either AoA or AoN networks
(here, in AoA: 7 arcs – 7 activities)

Intuitively **Transitive**

Detection Method: Transformation

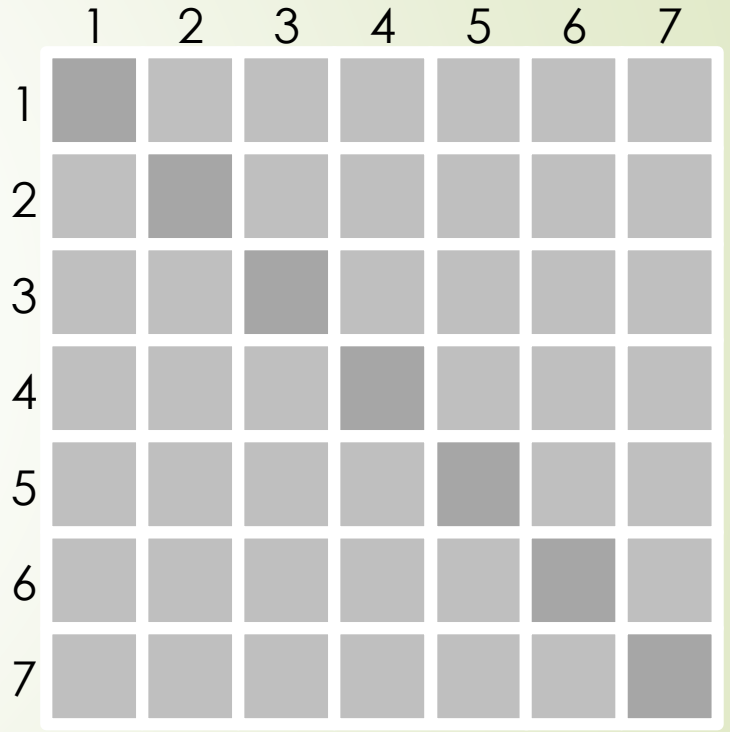




Transitive Precedence Relation

$row \times column =$ ■ means
"row" ***must occur before*** "$column$"

If *neither* activity a or b *precedes the other* they can execute *concurrently*



Parallel Relation (matrix form)

$row \times column =$ ■ means
"row" can execute in parallel with "$column$"

Detection Method: Identification

Parallel Relation Matrix → ***adjacency*** → Simple Undirected Graph

Set of activities, such:

Any two different activities may execute in parallel

+

There is no activity outside the set which can occur in parallel to any from the set

↓

The largest possible set whose activities can execute in parallel

Parallel Set

Set of nodes, such:

Any two different nodes are directly connected

+

There is no node outside the set which is directly connected to all in the set

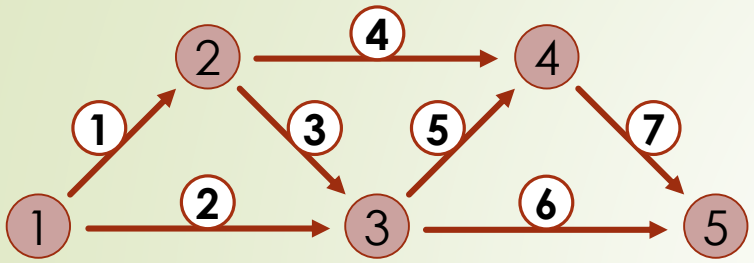
↓

The largest possible set of nodes which are connected to each other

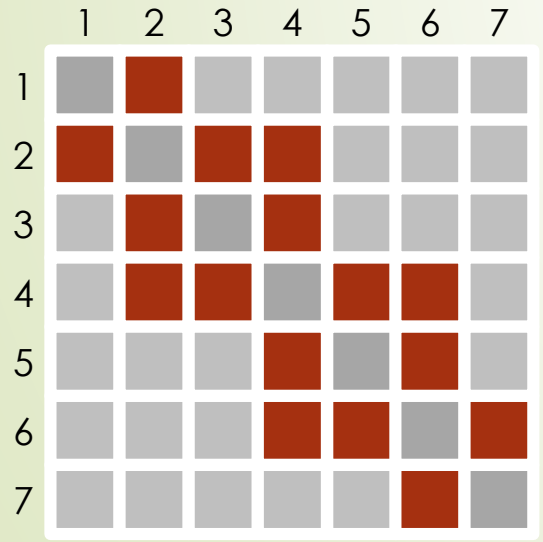
Maximal Clique

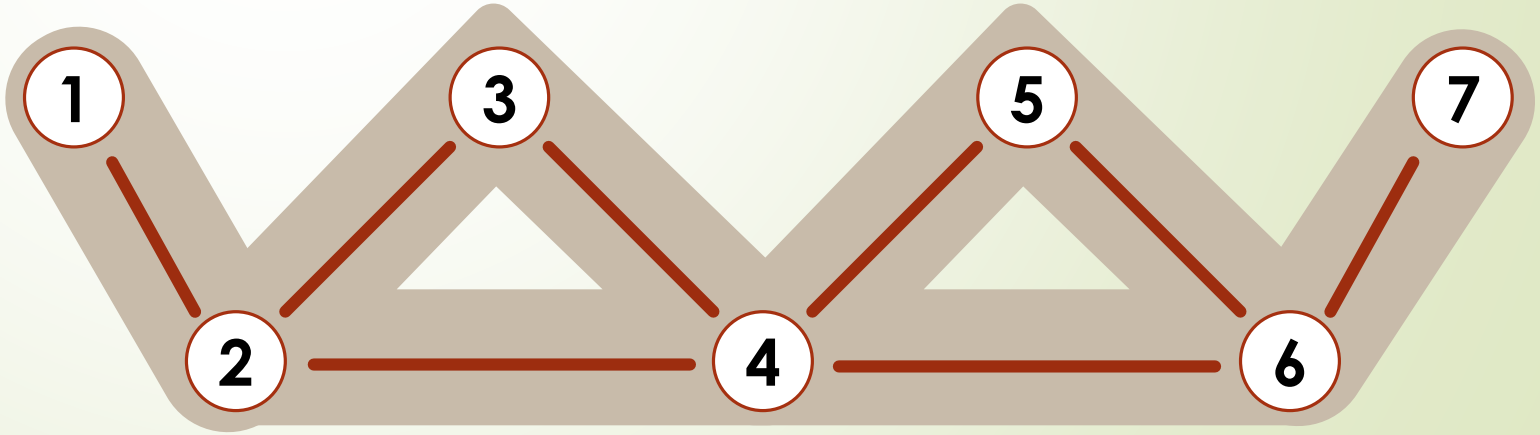
All Parallel Sets ⇋ All Maximal Cliques

Detection Method: Identification

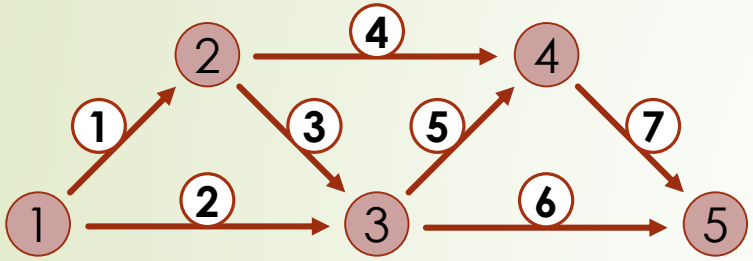


Parallel Sets:	
{1, 2}	{4, 5, 6}
{2, 3, 4}	{6, 7}



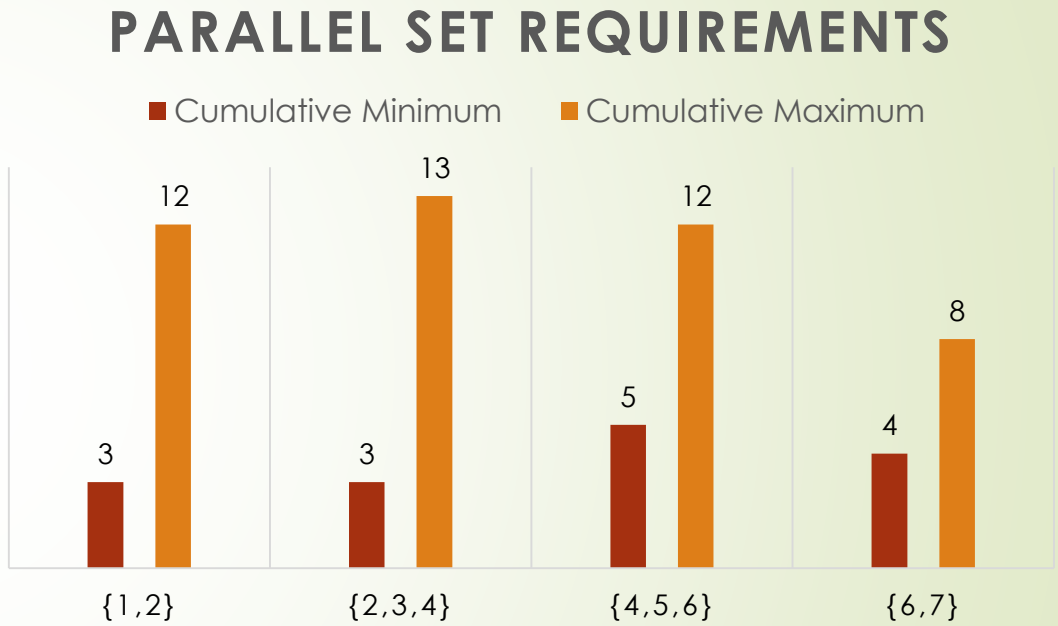


Application: Resource Availability Constraint



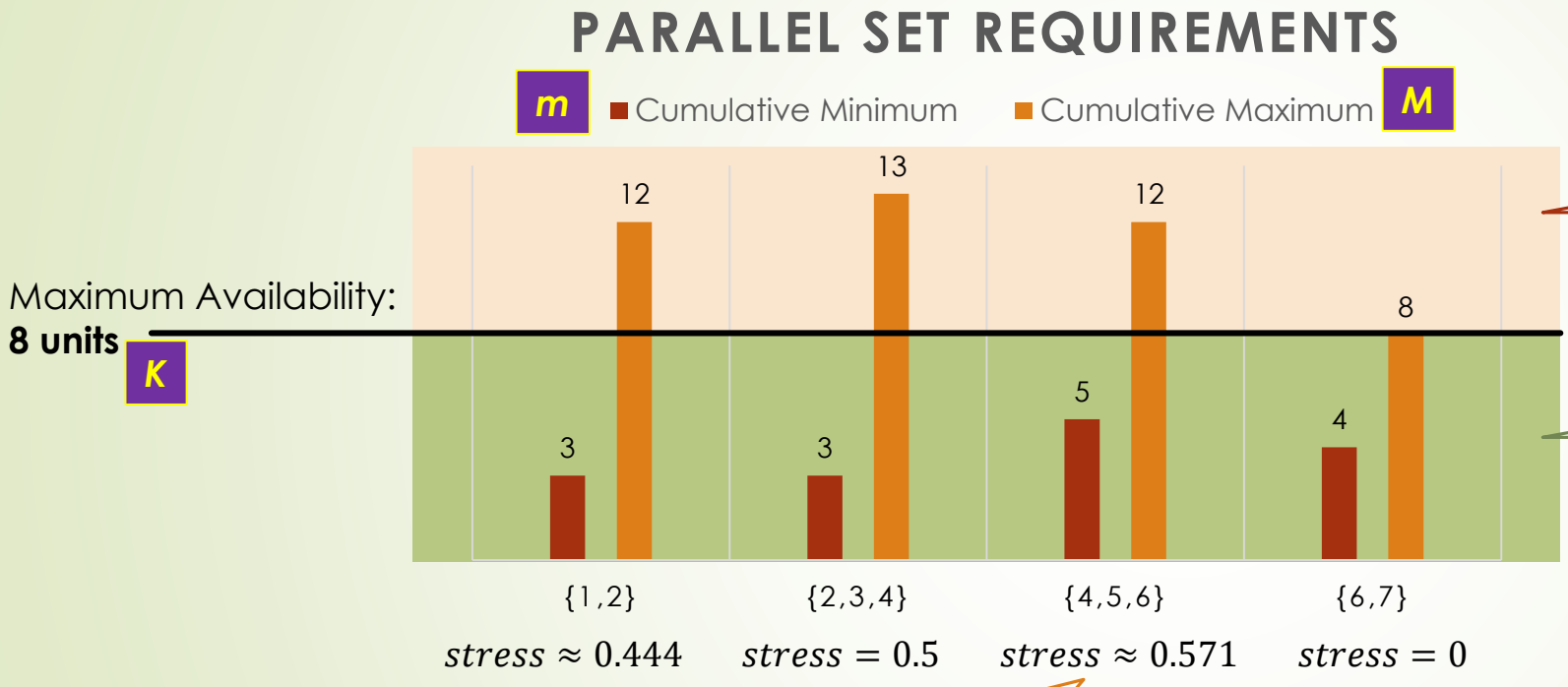
Multimodal Activities:

1. Min = 2; Max = 7
2. Min = 1; Max = 5
3. Min = 1; Max = 4
4. Min = 1; Max = 4
5. Min = 2; Max = 3
6. Min = 2; Max = 5
7. Min = 2; Max = 3



- With 13 units or more, no resource constraint is imposed
- With 5 units or more, any set is supported
- With less than 3 units, no concurrency will be possible

Application: Resource Stress



Availability **prohibits** concurrency scenarios

Availability **allows any** concurrency scenarios

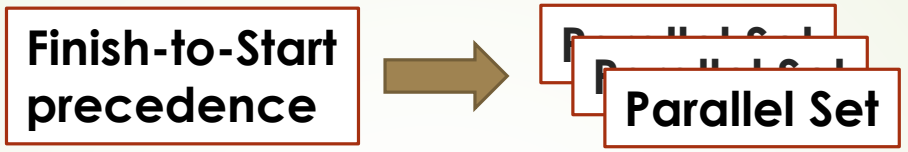
The maximu $\frac{8-8}{8-4} = 0$ .571 57%

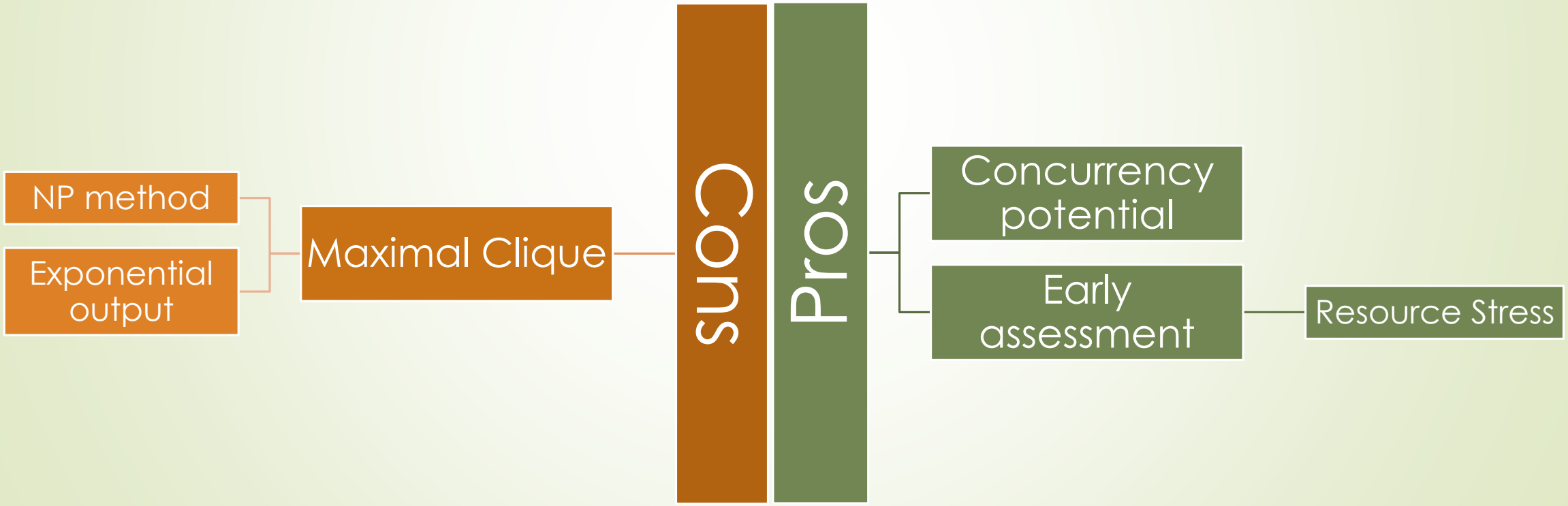
$$stress = \begin{cases} \frac{M-K}{M-m} & m \leq K \leq M \\ 0 & K < M \\ 1 & K < m \end{cases}$$

Resource Stress (of a parallel set)

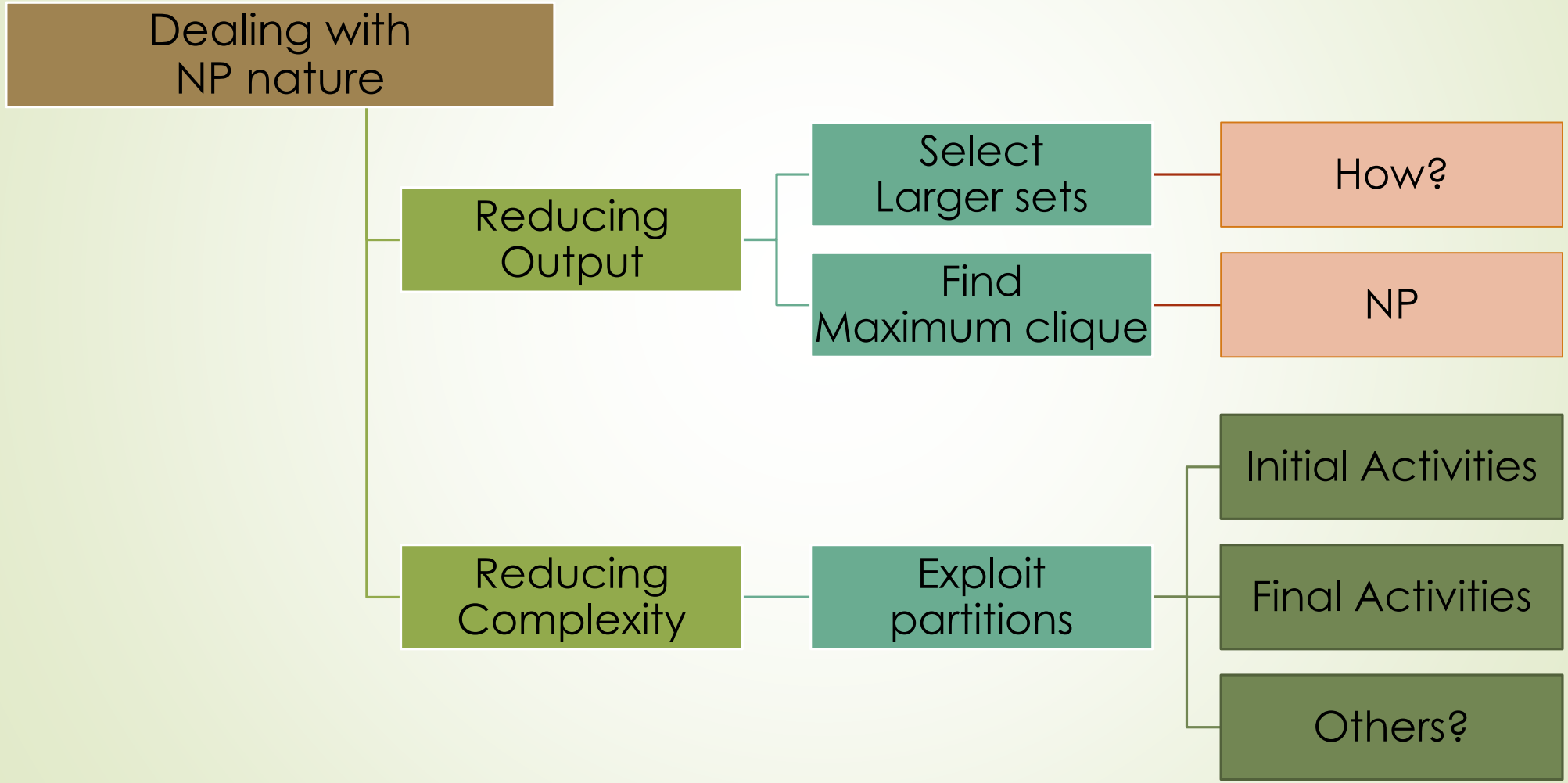
The probability of the cumulative requirement being greater than the maximum availability

Conclusions & Future Research





Conclusions & Future Research



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