Adaptive Resource Allocation in Multimodal Activity Networks

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Topics

1 - Problem Definition

2 - The DP Model

3 - Example Network

4 - Results

5 - Sensitivity Analysis

6 - Future Research

1. Problem Definition

- Given a multimodal activity network under stochastic conditions, we want to optimize the resource allocation to minimize cost

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Optimization via DP

1. Problem Definition

- Network Representation
  - Activity-on-arc
- Activity (A - set of activities)
  - Multimodal
- Work Content
  - Random $\rightarrow$ $\boxed{W_a \sim \exp(\lambda_a)}$
- Resource Allocation
  - Lower and upper bounds on allocations

$$\rightarrow \quad \boxed{0 \le l_a \le x_a \le u_a}$$

1. Problem Definition

- Duration
  - $Y_a = W_a / X_a$
- Resource Cost

  Assumed quadratic in allocation for the duration
  - $RC_a \propto x_a^2\, Y_a = x_a\, W_a$
- Due date
  - T
- Tardiness Cost
  - $TC = c_L \max\{0, \Upsilon_n - T\}$

1. Problem Definition

- Goal
  - Determine the resource allocation vector X_a, such that the total expected cost is minimized

$$\boxed{\min_{x} E\left\{ \sum_{a \in A} x_a . W_a + c_L \,.\, \max\left\{ 0 \,,\, \Upsilon_n - T \right\} \right\}}$$

2. The DP Model

- D - subset of decision variables
- F - subset of activities to be 'conditioned upon'

$\downarrow$

each 'udc' of the network contains exactly one decision variable

2. The DP Model

- Stage of DP
  - epoch of decision on $x_a \in D$
- At each stage
  - optimization over one decision variable
- Nº of stages
  - $K = | D | = | A | - | F |$
- State
  - $s_k = (t_{i_1}, \ldots, t_{i_r})$

2. The DP Model

- Stage “reward”
  - For last stage
    - resource cost + tardiness cost
  - For other stages
    - resource cost
- Stage numbering
  - Backwards
    - Stage K: K stages to go to complete the project

2. The DP Model

- DP transformation function (for stages 2..K)

$$f_k(s_k|F) = \min_{x_k \in D} E\{ x_k.W_k + E\, f_{k-1}(s_{k-1}|F)\} \quad (1)$$

- Deconditioning

$$f(s_k=0) = \min_{F} f_k(s_k|F)$$

- Solution via DP
  - policy that prescribes the optimal resource allocation under every conceivable state of the project as it progresses over time

2. The DP Model

- Application of the DP model
  - Process used to select set F
    1. Determine the longest path in the network
    2. The activities on the longest path will be decision variables (set D)
    3. The others will be the activities to be fixed (set F)
  - Resource cost of fixed variables

$$\boxed{rcf = E \sum_{i \in F} x_i . W_i = \sum_{i \in F} x_i . E\,(W_i)}$$

2. The DP Model

- First stage

$$\boxed{f_1(s_1|F) = \min_{x_{[1]} \in D} E\{\, x_{[1]}.W_{[1]} + rcf + c_L.\, E(U) \,\}}$$

where $U = \max\{\, 0\,,\, \Upsilon_n - T \,\}$

- Next stages
  - Apply expression (1), until last node is reached

2. The DP Model

- Last steps of the DP model application
  - This process $\rightarrow$ best allocation to the first activity $x_{[k]}$
  - Repeat procedure for all possible fixed allocations to the activities in the set F

    $\downarrow$
  - Resource allocation to the activities emanating from node 1

2. The DP Model

- – Policy afterwards: depends on the state of the process

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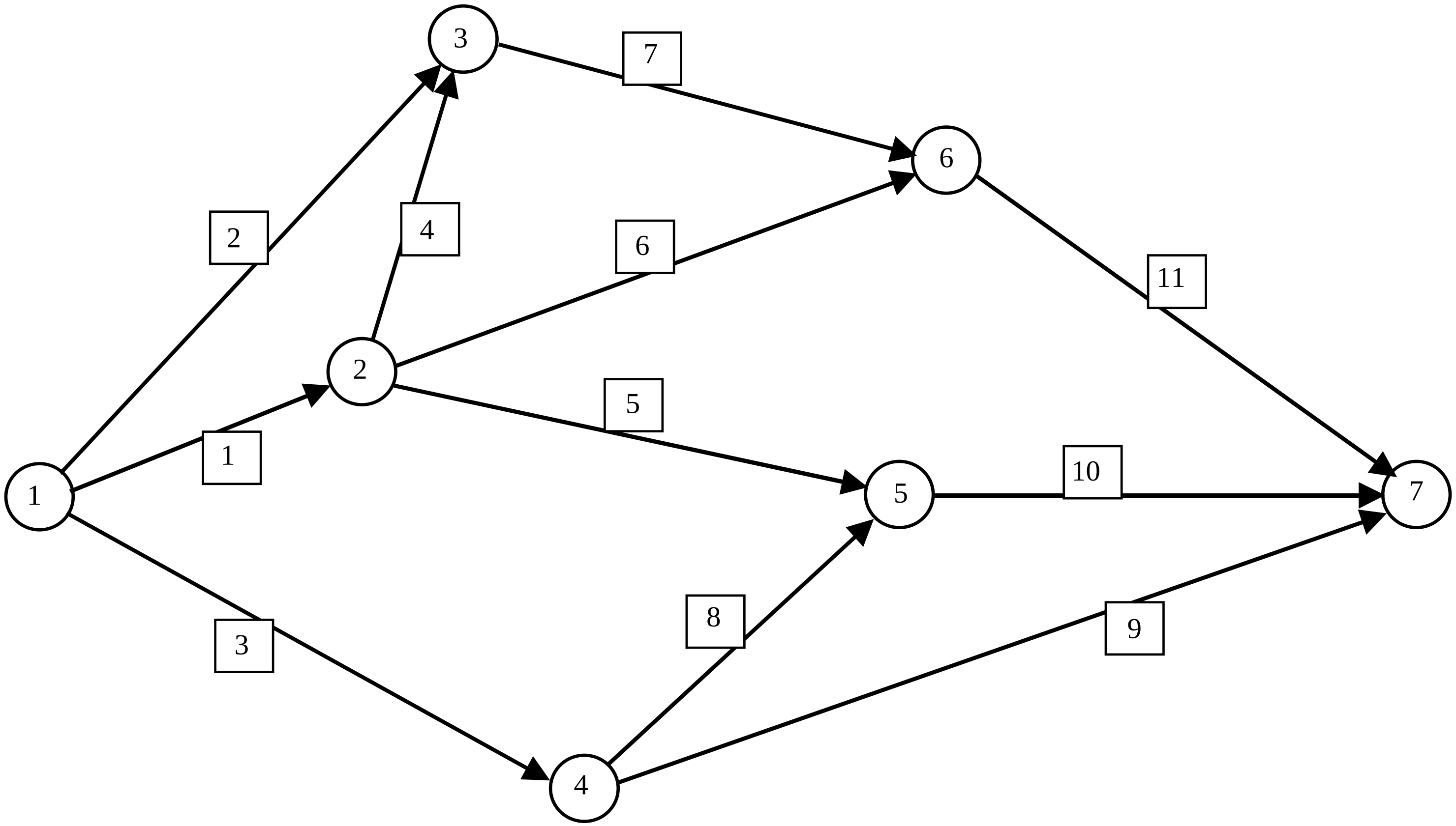
- – Adaptive nature of the DP approach: later allocations must await the realization of preceding activities as the project evolves over time

3. Example Network



3. Example Network

Activity a	Parameter λ_a	Expected Work Content
1	0.10	10.00
2	0.12	8.33
3	0.05	20.00
4	0.08	12.50
5	0.20	5.00
6	0.04	25.00
7	0.03	33.33
8	0.04	25.00
9	0.024	41.67
10	0.15	6.67
11	0.16	6.25

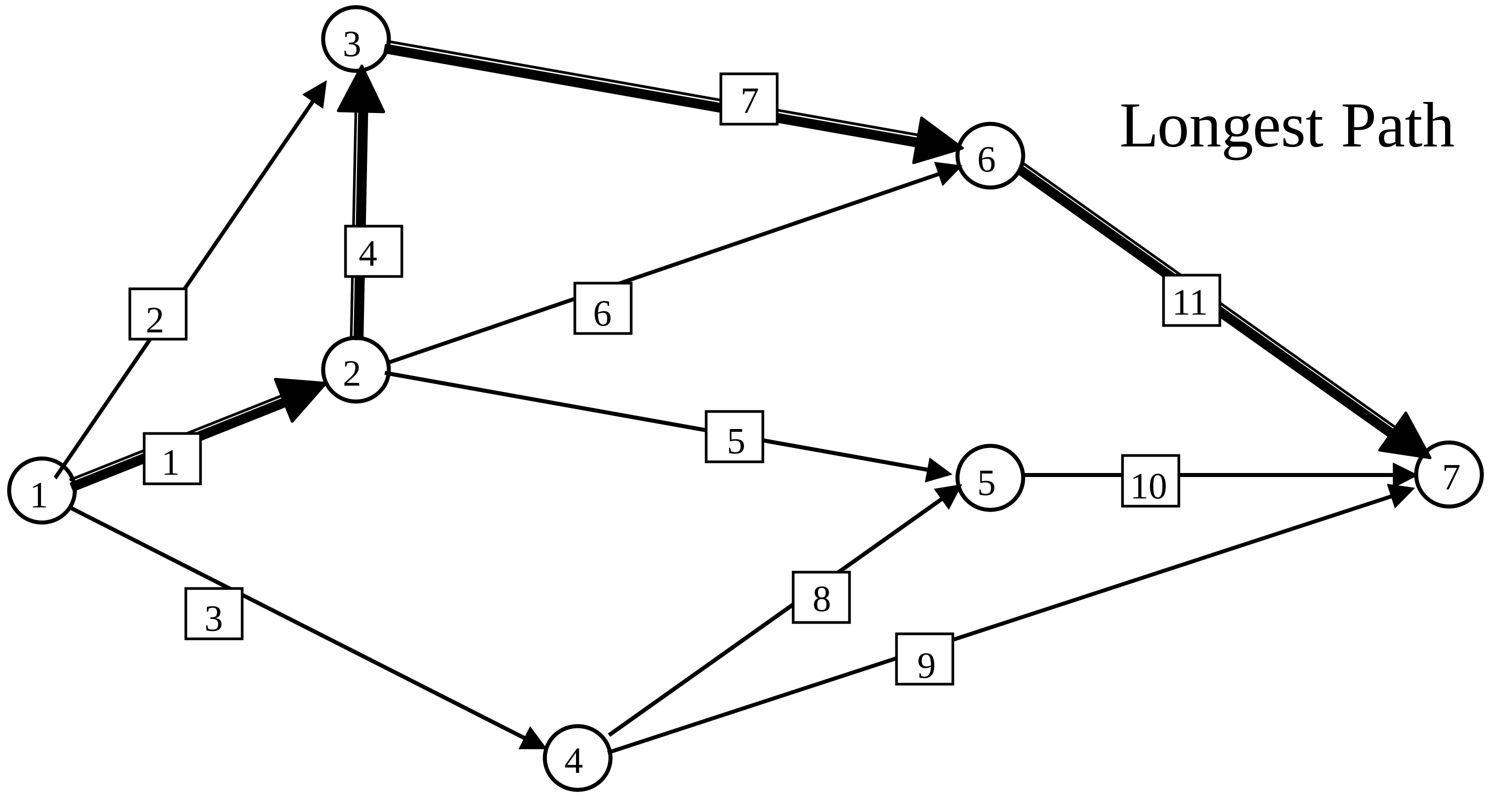
$T=65$

$C_L=5$

$0.5 \leq x_a \leq 1.5$

Unit: weeks

3. Example Network



$D = \{x1, x4, x7, x11\}$ $F=\{\hat{x}2, \hat{x}3, \hat{x}5, \hat{x}6, \hat{x}8, \hat{x}9, \hat{x}10\}$

3. Example Network

- x discretized: {0.5, 0.75, 1, 1.25, 1.5}
- W: discretized in 4 values

  p.e. W1 ~ exp(0.1) $\rightarrow$ {1.44, 4.9, 10.4, 19.83}

- With F fix at:

  $\{\hat{x}2, \hat{x}3, \hat{x}5, \hat{x}6, \hat{x}8, \hat{x}9, \hat{x}10\} = \{0.5, 1, 0.5, 1, 1, 0.5, 0.5\}$

- $\text{rcf} = \sum_{a \in F} x_a . E(W_a) = 100.83$

3. Example Network

- DP iterations
  - Stage 1

$$f_1(t_2,t_4,t_6 \mid F) = rcf + \min_{x_{11}} E\{ x_{11}.W_{11} + c_L.\ E(U) \}$$

$$U = \max\{ 0 , \Upsilon_7 - T \}$$

$$\Upsilon_7 = \max\{ t_6+W_{11}/x_{11}, \max\{ t_4+W_9/\hat{x}_9, W_{10}/\hat{x}_{10}+ \max\{ t_2+W_5/\hat{x}_5, t_4+W_8/\hat{x}_8 \} \} \}$$

3. Example Network

– Stage 2

$$f_2(t_2,t_3,t_4 \mid F) = \min_{x_7} E \{ x_7.W_7 + E [f_1 (t_2,t_4, \Upsilon_6)] \}$$

$$\Upsilon_6 = \max \{ t_2+W_6/\hat{x}_6, t_3+W_7/x_7 \}$$

– Stage 3

$$f_3(t_2,t_4 \mid F) = \min_{x_4} E \{ x_4.W_4 + E [f_2 (t_2, \Upsilon_3,t_4)] \}$$

$$\Upsilon_3 = \max \{ t_2+W_4/x_4, W_2/\hat{x}_2 \}$$

3. Example Network

– Stage 4

$$f_4(t_1=0 \mid F)= \min_{x_1} E \{ x_1.W_1 + E [f_3 (\Upsilon_2, \Upsilon_4)] \}$$

$$\Upsilon_2 = W_1/x_1 \qquad \Upsilon_4 = W_3/\hat{x}_3$$

4. Results

- First result:

  $x_1|F=0.75$ at a total expected cost of 383.86

- Removing conditioning on F
  - Evaluate the expected cost and the optimal value of the decision variable for every combination of the fixed variables

$$\downarrow$$

$5^7 = 78125$ enumerations

4. Results

- Simplification: enumerate only over 3 values

$\downarrow$

3^7=2187 enumerations

- Final result

$(x_1^*, x_2^*, x_3^*) = (0.75, 0.5, 1.5)$

at a total expected cost of 239.76

4. Results

- We also have
  - Complete vector of allocations for the fixed activities that yielded the optimum

$$\{\hat{x}_2,\hat{x}_3,\hat{x}_5,\hat{x}_6,\hat{x}_8,\hat{x}_9,\hat{x}_{10}\}=\{0.5,1.5,0.5,1.0,1.0,1.5,1.0\}$$

  - Corresponding optimal policies at all remaining stages of the project

4. Results

- The dynamic behaviour of the process at first stage
  1. If activity 1 completes first $\rightarrow$ t_2 will be known $\rightarrow$ x_4^*
  2. If activity 2 completes first $\rightarrow$ dormant
  3. If activity 3 completes first $\rightarrow$ initiate activities 8 and 9

5. Sensitivity Analysis

- Goal
  - Test the sensitivity of the total cost to small variations in the variables
- One-at-a-time variation
  - For each x_k^* evaluate the performance of the project cost at each of the 'neighbouring' values

5. Sensitivity Analysis

- Possible outcomes of the cost

  1. Monotone increasing: decrease x^* if possible
  2. Monotone decreasing: increase x^* if possible
  3. Is 'V' shaped: test at both sides of x^*
  4. Is 'Λ' shaped: test at both sides of x^*

Sensitivity Analysis

Table 1

Expected Value		Best x2	Best x3	Best x5	Best x6	Best x8	Best x9	Best x10
x's	0,5	239,76	293,92	239,76	288,8	290,97	371,1	243,59
	1	242,15	242,82	242,26	239,76	239,76	249,8	239,76
	1,5	246,32	239,76	244,76	252,23	246,52	239,76	241,65
		0,5	1,5	0,5	Tab. 2	Tab. 3	1,5	Tab. 4

Table 2

Expected Value		1st	2nd
x6	0,625		250,45
	0,75	234,26	234,26
	0,875		236,64
	1	239,76	
	1,25	246,01	
		0,75	0,75

Table 3

Expected Value		1st
x8	0,75	250,49
	1	239,76
	1,25	242,15
		1

Table 4

Expected Value		1st
x10	0,75	240,12
	1	239,76
	1,25	240,51
		1

Conclusion:

Before Sensitivity Analysis:

Best Expected Value=239,76

(x1, x2, x3, x5, x6, x8, x9, x10)

(0.75, 0.5, 1.5, 0.5, 1.0, 1.0, 1.5, 1.0)

After Sensitivity Analysis:

Best Expected Value=234,26

(x1, x2, x3, x5, x6, x8, x9, x10)

(1.0, 0.5, 1.5, 0.5, 0.75, 1.0, 1.5, 1.0)

5. Sensitivity Analysis

- Result

$\{\ x_1^*,\ x_2^*,\ x_3^*,\ x_5^*,\ x_6^*,\ x_8^*,\ x_9^*, x_{10}^*\} =$
$\{1.0, 0.5, 1.5, 0.5, 0.75, 1.0, 1.5, 1.0\}$

with expected cost = 234.26

6. Current Research

- Approximations under investigation
  - Switching the Order (SO) Approach
    - Replace the search for the optimum of the expected value with the search of the expected value of the optima
  - Activity Aggregation (AA) Approach
    - Combination of two or more activities into a large ‘aggregate activity’

6. Current Research

- Apply other compu-search approaches
  - Use of various techniques as:
    - Monte Carlo Simulation
    - CPM evaluation
    - Global optimization

‘Electromagnetism Algorithm’ designed by Birbil and Fang (2000)

References

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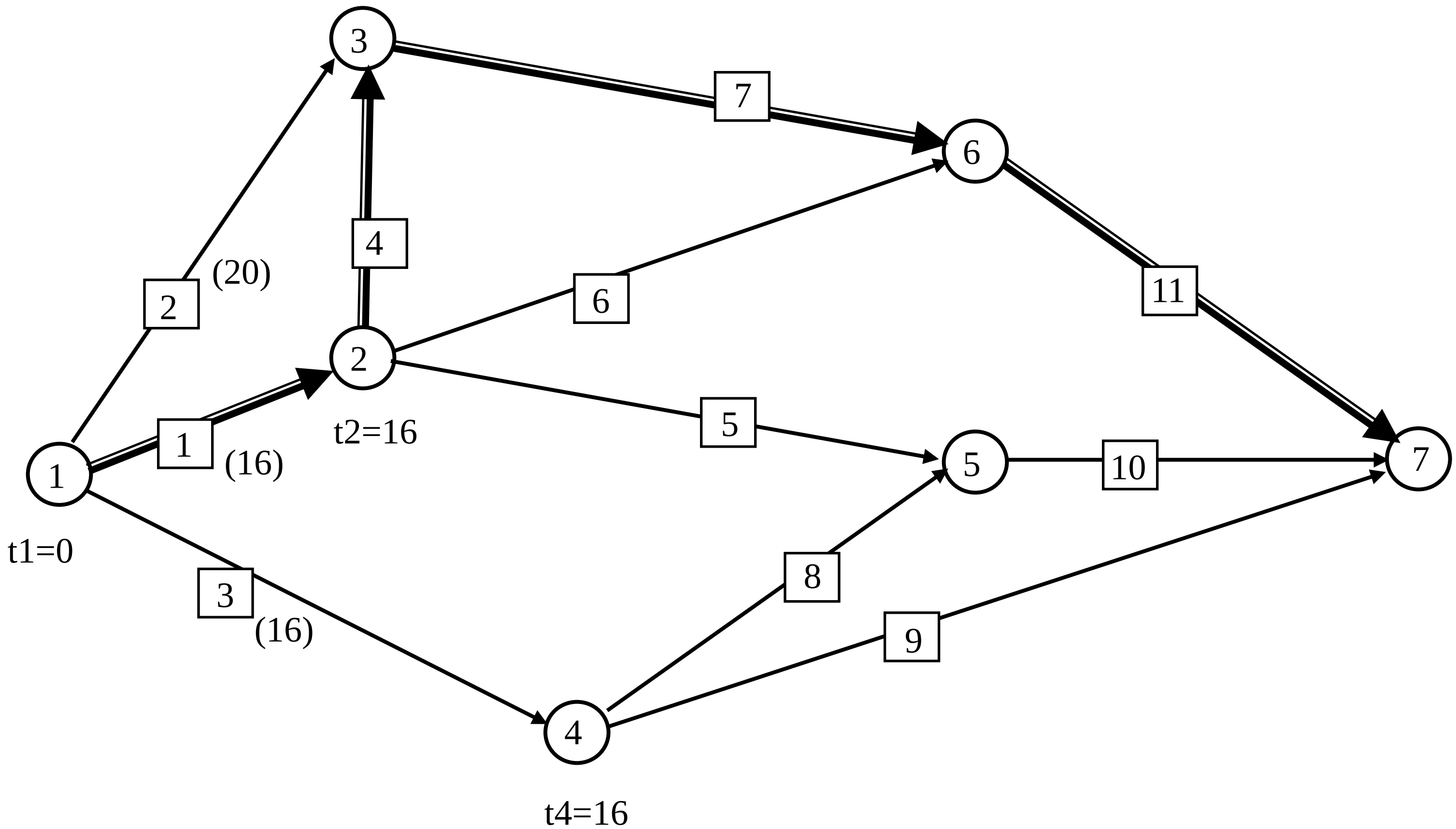
2. Birbil, S.I. and Fang, S-C. (2000), ``Electromagnetism for global optimization'', submited to Journal of Global Optimization.
3. Colby, A.H. and Elmaghraby, S.E. (1984). ``On the Complete Reduction of Directed Acyclic Graphs'', OR Report No. 197, May, Graduate Program in Operations Research, NCSU.
4. Davis, E.W. (1973). ``Project scheduling under resource constraints - an historical review and categorization of procedures,'' AIIE Trans 5, 297-313.
5. Demeulemeester, E. and Herroelen, W.S. (1995). ``New benchmark results for the resource-constrained project scheduling problem,'' Res. Report NR 9521, Department of Applied Economics, Catholic Univ. of Leuven, Leuven, Belgium.

Appendix: Scenario Illustration

t=0

Act
1
2
3

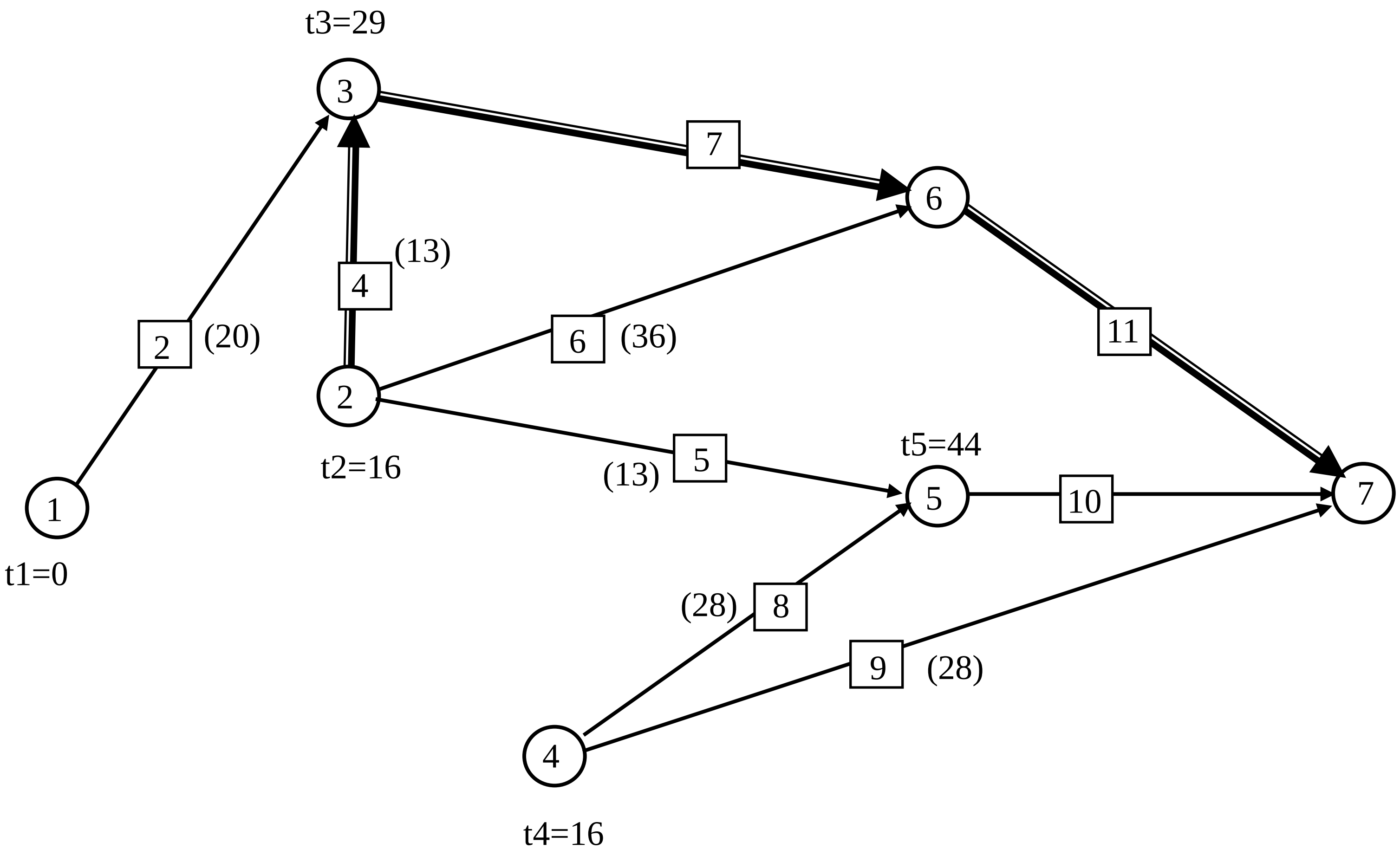


From DP: x1*=1, x2*=0.5, x3*=1.5

If w1=16, w2=10, w3=24 → Y1=16, Y2=20, Y3=16 → t2 =16, t4 = 16

t=16

Act
2
4
5
6
8
9



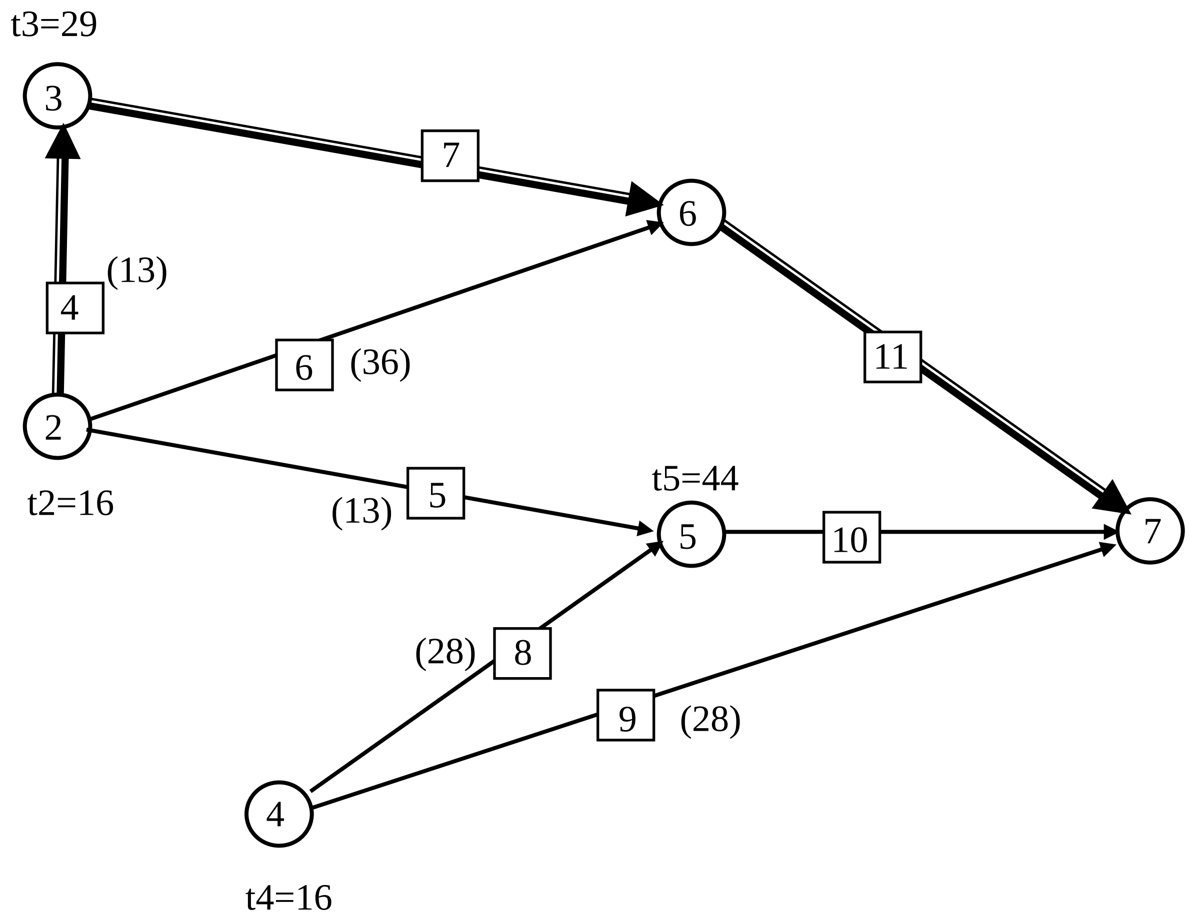
From DP: $x_4^*=1$, $x_5^*=0.5$, $x_6^*=0.75$, $x_8^*=1$, $x_9^*=1.5$

If $w_4=13$, $w_5=6.5$, $w_6=27$, $w_8=28$, $w_9=42$ → $Y_4=13$, $Y_5=13$, $Y_6=36$, $Y_8=28$, $Y_9=28$ →

$t_3=29$, $t_5=44$

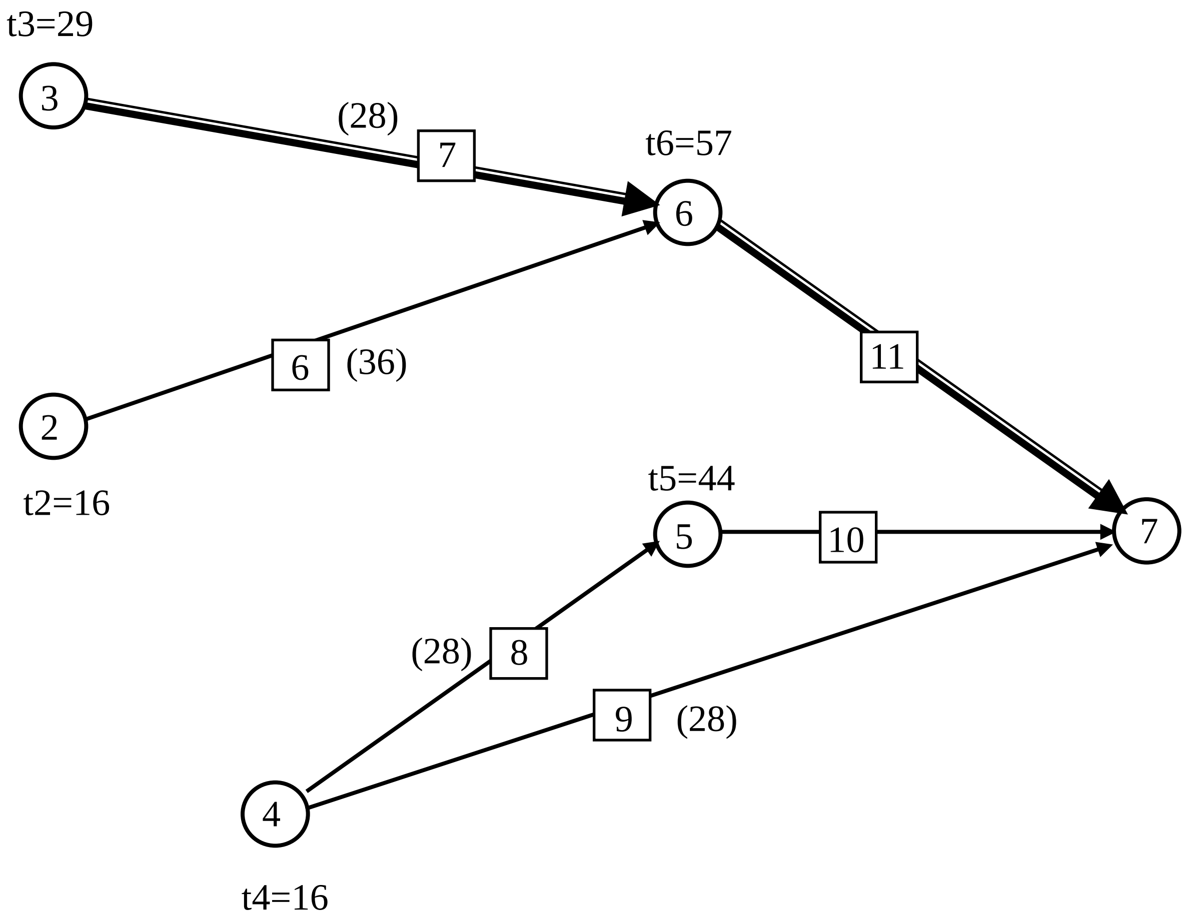
t=20

Act
4
5
6
8
9



t=29

Act
6
7
8
9

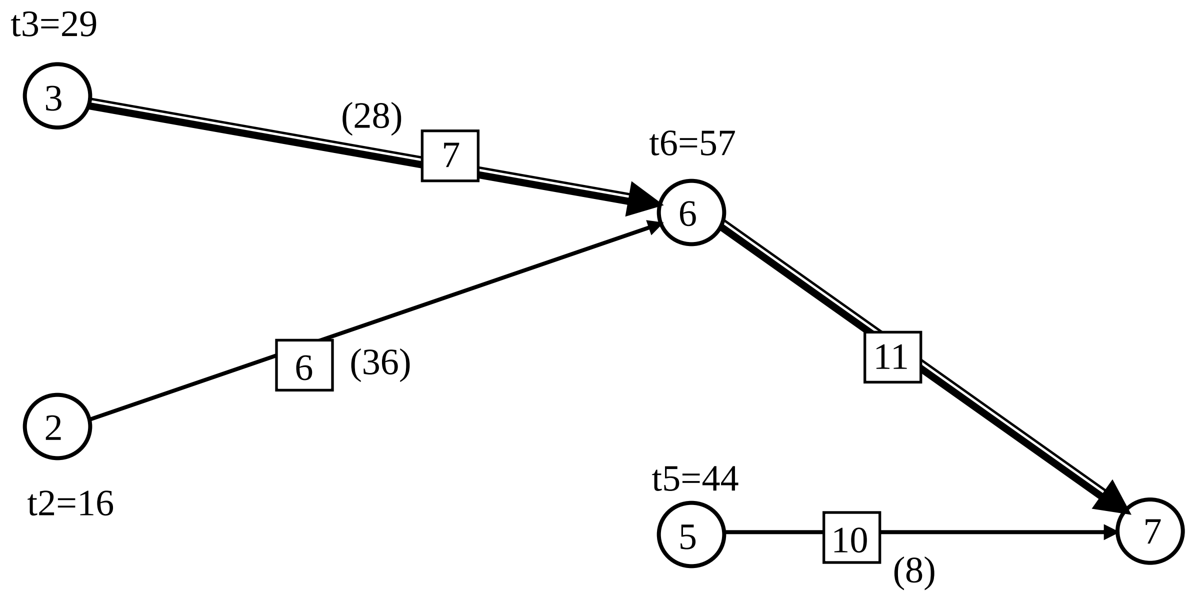


From DP: $x7^*=1.25$

If w7=35 → Y7=28 → t6=57

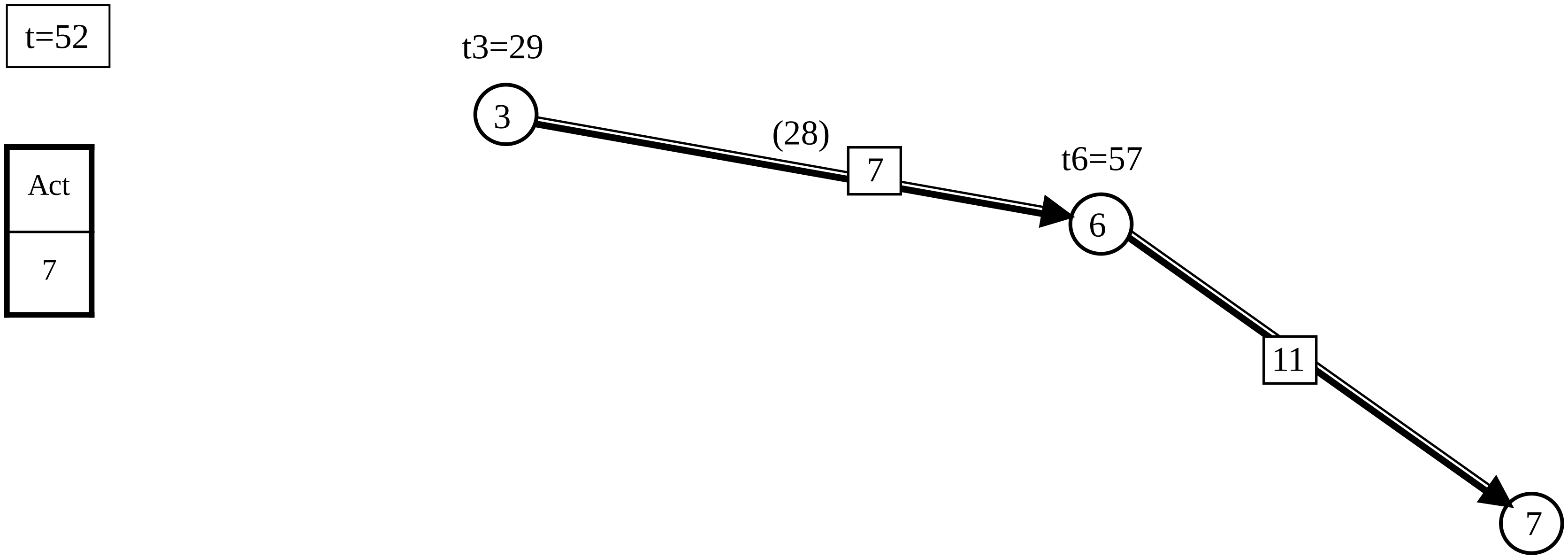
t=44

Act
6
7
10



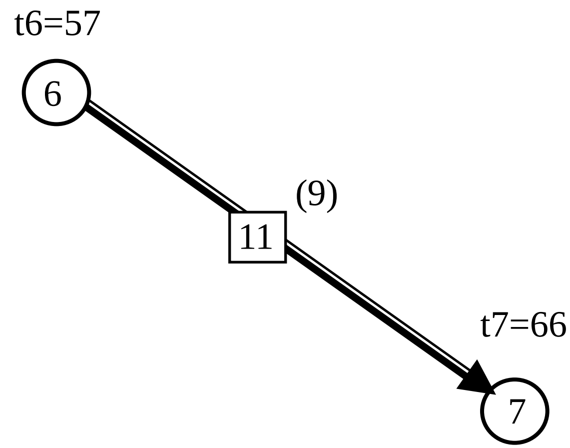
From DP: x10*=1

If w10=8 ⇒ Y10=8


t=52
Act
7
t3=29
3
(28)
7
t6=57
6
11
7

t=57

Act
11



From DP: x11*=1.25

If w11=11.25 → Y11=9 → t7=66

t=66

t7=66

End of project

7

T=65

tc=5*(66-65)=5

rc = xaWa ≃ 250 → Total cost = 255