Basic Approximations to an Adaptive Resource Allocation Technique to Stochastic Multimodal Projects

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Topics

1 – Problem Definition

2 – The DP Model

3 – Example Network

4 – Approximations Developed

5 – Examples and Results

6 – Conclusions and Future Research

1. Problem Definition (1/2)

- Given a multimodal activity network under stochastic conditions, we want to optimize the resource allocation to minimize cost

$$\downarrow$$

Optimization via DP

1. Problem Definition (1/2)

- Given a multimodal activity network under stochastic conditions, we want to optimize the resource allocation to minimize cost

$$\downarrow$$

Optimization via DP

1. Problem Definition (2/2)

- Goal
  - Determine the resource allocation vector X_a, such that the total expected cost is minimized

$$\boxed{\min_{x} E\left\{ \sum_{a \in A} x_a . W_a + c_L \, . \, \max\{0 \,,\, \Upsilon_n - T\}\right\}}$$

2. The DP Model (1/2)

- Process used to select set F
  1. Determine the longest path in the network
  2. The activities on the longest path will be the decision variables (set D)
  3. The others will be the activities to be fixed (set F)
- Resource cost of fixed variables

$$\boxed{rcf = E \sum_{i \in F} x_i . W_i = \sum_{i \in F} x_i . E\,(W_i)}$$

2. The DP Model (2/2)

- First stage

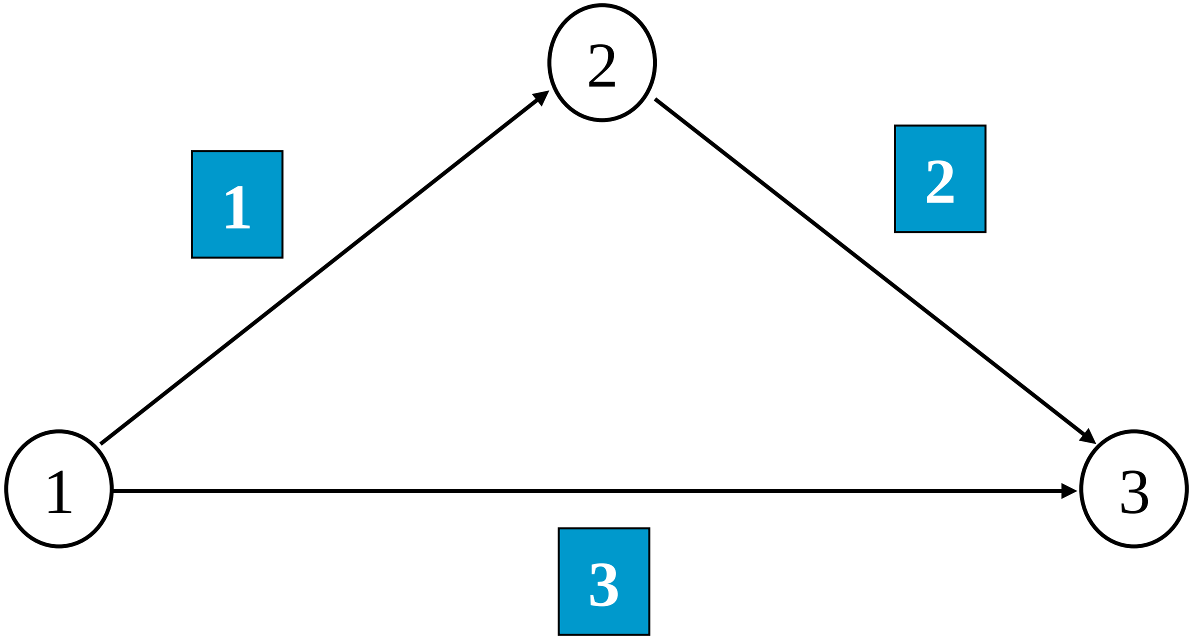
$$\boxed{f_1(s_1|F) = \min_{x_{[1]} \in D} E \{ x_{[1]}.W_{[1]} + rcf + c_L . E(U) \}}$$

where $U = \max \{ 0 , \Upsilon_n - T \}$

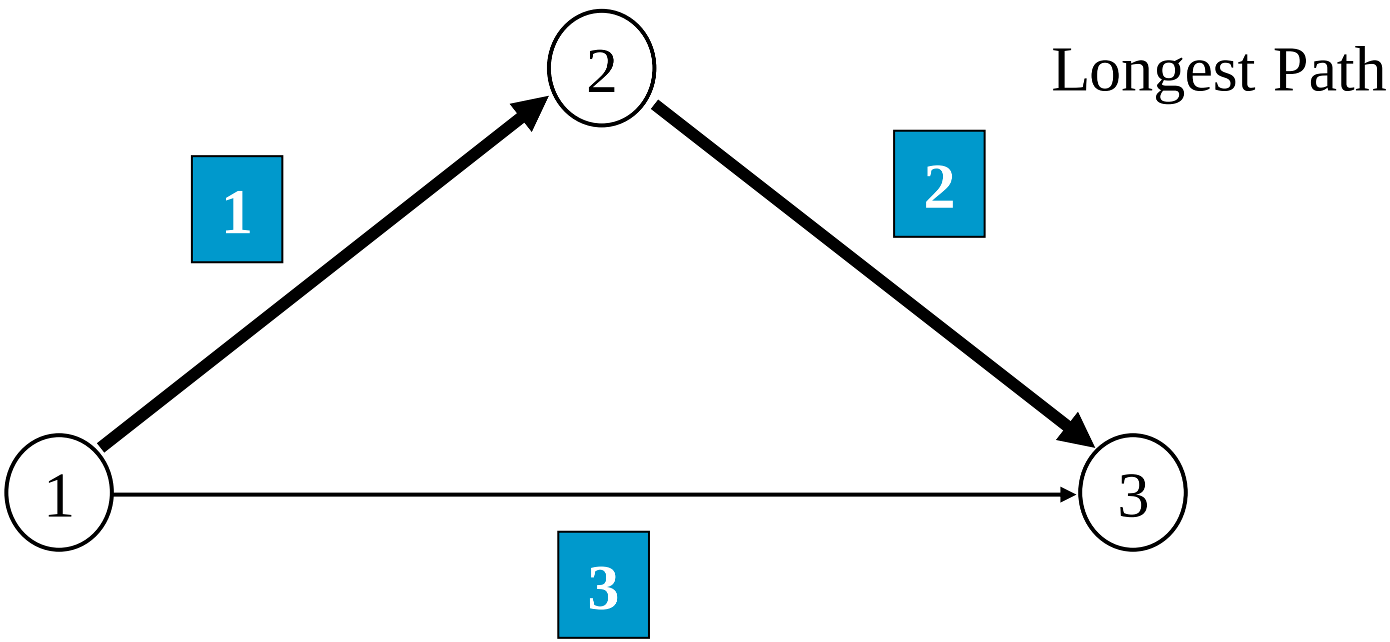
- Next stages

$$\boxed{f_k(s_k|F) = \min_{x_k \in D} E \{ x_k.W_k + E f_{k-1}(s_{k-1}|F) \}}$$

3. Example Network (1/5)



3. Example Network (1/5)



$D = \{x1, x2\}$ $F=\{x\hat{3}\}$

3. Example Network (2/5)

Activity a	Parameter λ_a	Expected Work Content
1	0.40	2.50
2	0.16	6.25
3	0.10	10.00

T=9

C_L=4

$0.5 \leq x_a \leq 1.5$

Unit: weeks

3. Example Network (3/5)

- X: discretized in {0.5, 0.75, 1, 1.25, 1.5}
- W: discretized in 4 values

  p.e. W3 ~ exp(0.1) $\rightarrow$ {1.37, 4.77, 10.0, 23.86}

- rcf = $x_3 \cdot E(W_3) = x_3 / 0.1$

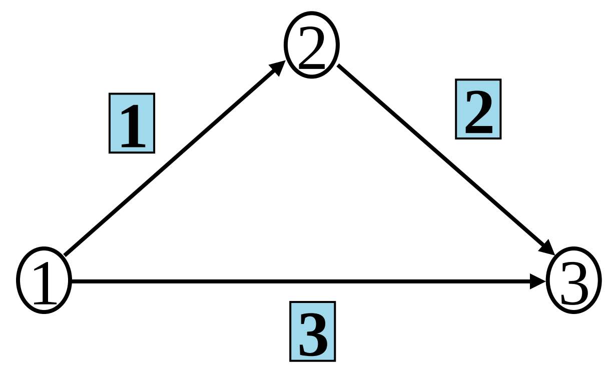
3. Example Network (4/5)

- DP iterations
  - Stage 1

$$f_1(\, t_2 \,|\, F=\{3\}) = rcf + \min_{x2} E\,\{\, x_2.W_2 + 4.\ E\,(U)\,\}$$

$$U = \max\{\, 0\ ,\ \Upsilon_3\text{-}T\,\}$$

$$\Upsilon_3 = \max\{\, t_2 + W_2/x_2,\ W_3/\hat{x}_3\,\}$$



  - Stage 2

$$f_2(\, t_1\!=\!0 \,|\, F=\{3\}) = \min_{x1} E\,\{\, x_1.W_1 + E\,[f_2\,(\Upsilon_2)]\,\}$$

$$\Upsilon_2 = W_1/x_1$$

3. Example Network (5/5)

- Computational Tests
  - Pentium III, 650 MHz, 128 MB
  - Windows Millennium

- Solution

  $(x_1^*, x_3^*) = (1.25, 1.5)$

  at a total expected cost of 35.97

  (running time: 0.22 s)

4. Approximations Developed (1/4)

- Approximation 1
  - Based on the DP Model
  - Work Contents of F → mean values
  - Example solution

    $(x_1^*, x_3^*) = (1.25, 1.0)$

    at a total expected cost of 27.65

    (running time: 0.19 s)

4. Approximations Developed (2/4)

- Approximation 2
  - Based on the DP Model
  - Work Contents of F and D → mean values
  - Example solution

    (x1*, x3*) = (1.25, 1.0)

    at a total expected cost of 21.81

    (running time: 0.12 s)

4. Approximations Developed (3/4)

- Approximation 3
  - Based on NLP
  - Work Contents of F and D → mean values
  - Example solution

    $(x1^*, x2^*, x3^*) = (0.8, 1.06, 1.11)$

    at a total expected cost of 19.79

    (running time: 0.1 s)

4. Approximations Developed (4/4)

- Approximation 3 (contd.)

Excel Solver

Activity a	Lambda a	Wa		xa	time ti				
1	0,4	2,5	**x1=**	0,8	0	: t1=0	**F(x)=**	19,79	
2	0,16	6,25	**x2=**	1,06	3,113	: t2=t1+W1/x1			
3	0,1	10	**x3=**	1,11	9,021	: t3=max(t1+W3/x3;t2+W2/x2)			
			delay u=	0,02					
			T=	9					
			cl=	4					

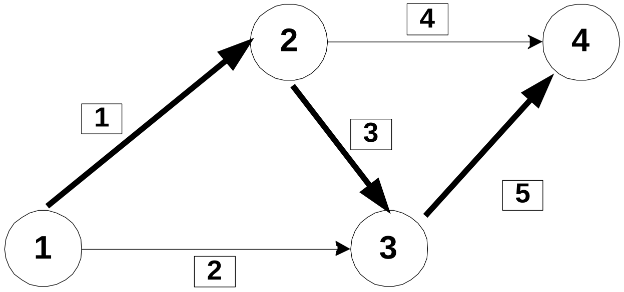
u=t3-T

F(x)=x1W1+x2W2+x3W3 +cl*max(0;u)

4. Examples and Results (1/9)

- Network 1

T=120
cL=8



Activity	1	2	3	4	5
Origin	1	1	2	2	3
Target	2	3	3	4	4
λ	0.02	0.03	0.04	0.024	0.025
x_{min}	0.5	0.5	0.5	0.5	0.5
x_{max}	1.5	1.5	1.5	1.5	1.5

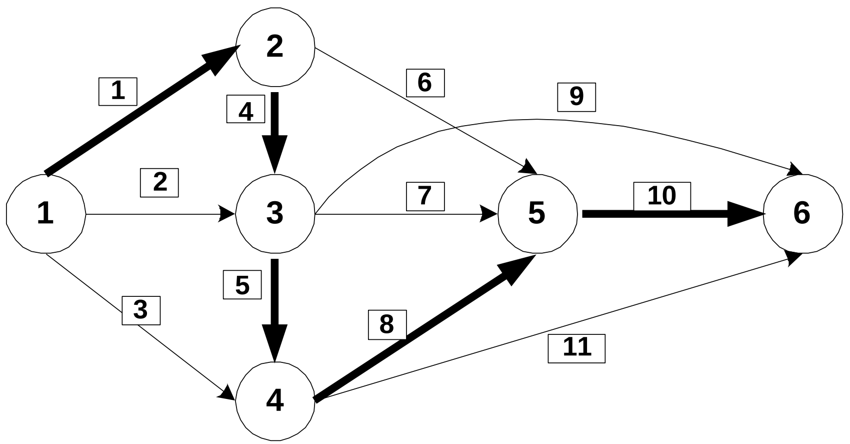
4. Examples and Results (2/9)

Network 1	DP Model	Approx. 1	Approx. 2	Approx. 3
x_1	1.0	1.0	1.25	1.03
x_2	1.0	1.0	0.5	0.51
x_3				0.87
x_4	1.5	1.0	0.5	0.59
x_5				0.96
EV(Cost)	304.62	279.14	148.79	152.82
Run Time	9.6 sec.	8.4 sec.	6 sec.	1 sec.

4. Examples and Results (3/9)

- Network 2

T=28
cL=8



Activity	1	2	3	4	5	6	7	8	9	10	11
Origin	1	1	1	2	3	2	3	4	3	5	4
Target	2	3	4	3	4	5	5	5	6	6	6
λ	0.1	0.09	0.4	0.2	0.3	0.08	0.4	0.2	0.1	0.3	0.3
x_{min}	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
x_{max}	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

4. Examples and Results (4/9)

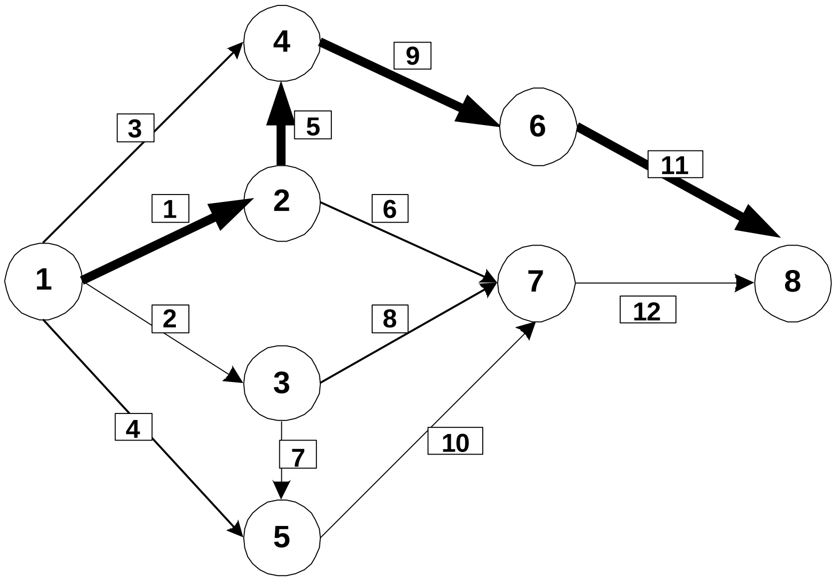
Network 2	DP Model	Approx. 1	Approx. 2	Approx. 3
x_1	1.25	1.25	0.75	1.15
x_2	1.0	1.0	1.0	0.8
x_3	0.5	0.5	0.5	0.5
x_4				0.95
x_5				1.16
x_6	1.0	0.5	1.0	0.82
x_7	0.5	0.5	0.5	0.5
x_8				0.72
x_9	1.0	0.5	1.0	0.9
x_{10}				0.82
x_{11}	1.0	0.5	0.5	0.5
EV(Cost)	106.76	75.04	58.94	58.75
Run Time	30 min.	13 min.	6 min.	1 sec.

4. Examples and Results (5/9)

- Network 3

T=47

cL=4



Activity	1	2	3	4	5	6	7	8	9	10	11	12
Origin	1	1	1	1	2	2	3	3	4	5	6	7
Target	2	3	4	5	4	7	5	7	6	7	8	8
λ	0.1	0.09	0.08	0.1	0.09	0.08	0.1	0.09	0.08	0.1	0.09	0.1
x_{min}	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
x_{max}	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

4. Examples and Results (6/9)

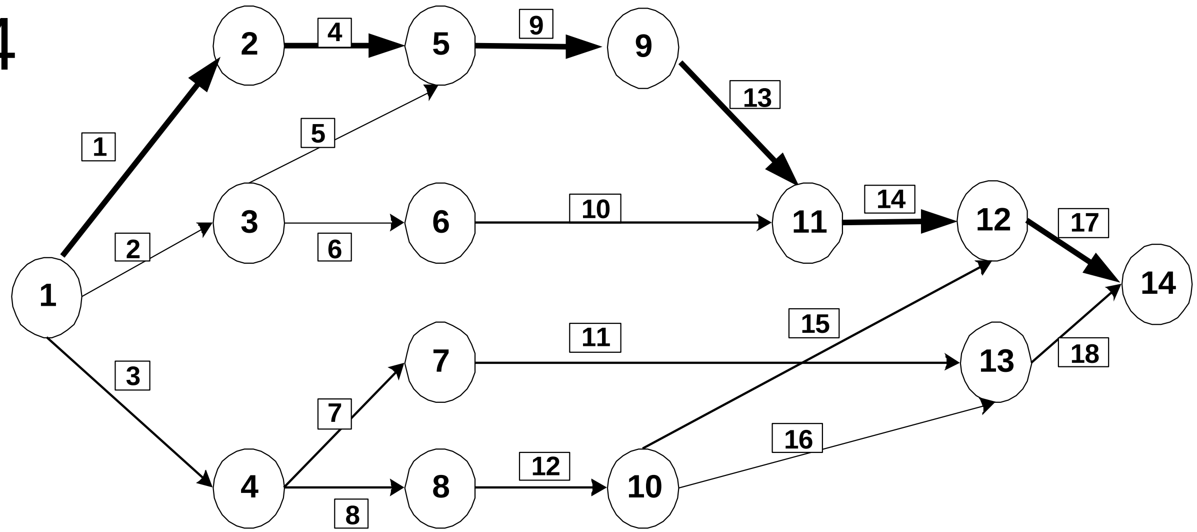
Network 3	DP Model	Approx. 1	Approx. 2	Approx. 3
x_1	1.25	1.25	0.75	0.93
x_2	1.0	0.5	1.0	0.81
x_3	1.0	1.0	0.5	0.55
x_4	0.5	0.5	0.5	0.5
x_5				0.93
x_6	1.0	0.5	0.5	0.5
x_7	1.0	0.5	1.0	0.82
x_8	0.5	0.5	0.5	0.5
x_9				0.97
x_{10}	1.0	0.5	1.0	0.82
x_{11}				0.97
x_{12}	1.5	1.0	1.0	1.15
EV(Cost)	182.91	117.92	103.26	91.53
Run Time	23 h.	7 h.	3 h.	1 sec.

4. Examples and Results (7/9)

- Network 4

T=110

cL=10



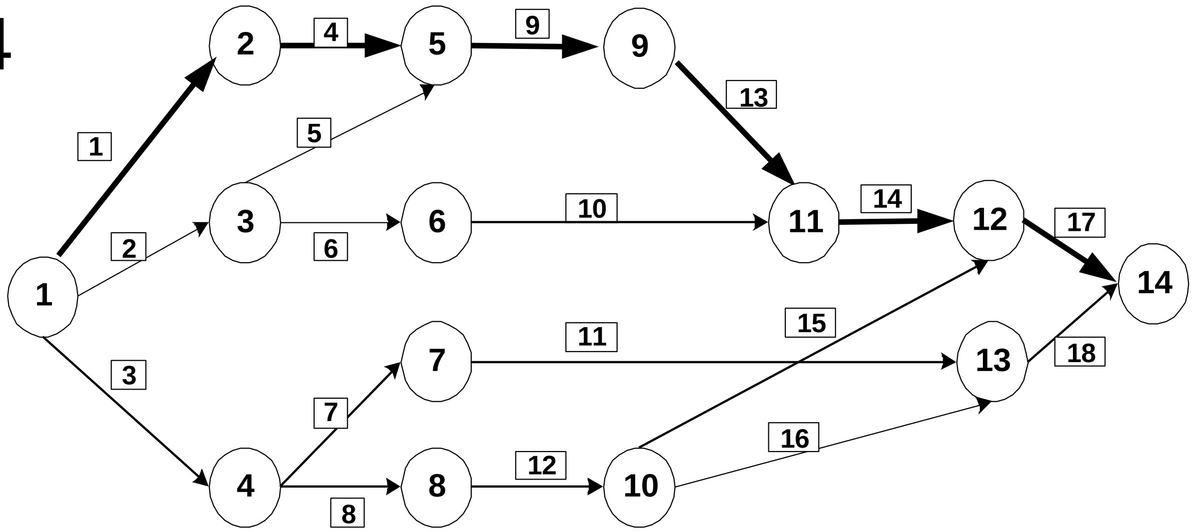
Activity	1	2	3	4	5	6	7	8	9
Origin	1	1	1	2	3	3	4	4	5
Target	2	3	4	5	5	6	7	8	9
λ	0.06	0.04	0.1	0.07	0.08	0.04	0.08	0.2	0.07
x_{min}	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
x_{max}	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

4. Examples and Results (8/9)

- Network 4

T=110

cL=10



Activity	10	11	12	13	14	15	16	17	18
Origin	6	7	8	9	11	10	10	12	13
Target	11	13	10	11	12	12	13	14	14
λ	0.05	0.08	0.07	0.09	0.09	0.05	0.09	0.04	0.06
x_{min}	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
x_{max}	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

4. Examples and Results (9/9)

Network 4	DP Model	Approx. 1	Approx. 2	Approx. 3
x_1	0.75	1.25	0.5	0.69
x_2	1.25	1.25	0.75	1.3
x_3	1.25	0.75	0.75	0.5
x_4				0.67
x_5	0.75	0.75	0.75	0.5
x_6	1.25	1.25	0.75	1.29
x_7	0.75	0.75	0.75	0.5
x_8	1.25	0.75	0.75	0.5
x_9				0.66
x_{10}	1.25	0.75	0.75	1.14
x_{11}	1.25	0.75	0.75	0.5
x_{12}	0.75	0.75	0.75	0.59
x_{13}				0.73
x_{14}				1.02
x_{15}	0.75	0.75	0.75	0.58
x_{16}	1.25	0.75	0.75	0.55
x_{17}				1.5
x_{18}	0.75	0.75	0.75	0.56
EV(Cost)	339.07	271.38	210.19	144.07
Run Time	5 days	9 h.	2 h.	1 sec.

5. Conclusions and Future Research (1/2)

- Conclusions
  - DP model
    - ✓Exact model
    - ✓Larger networks → very heavy
  - Approximations
    - ✓The running time reduces
    - ✓EV(cost) tends to be lower
    - ✓*x*'s: stable values → use them when implementing the project

5. Conclusions and Future Research (2/2)

- Future Research
  - Activity Aggregation (AA) Approach
  - Apply other compu-search approaches to this problem (Monte Carlo Simulation, CPM evaluation, “Electromagnetism Algorithm” by Birbil and Fang)

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