
**ILS 2010**
**Third International Conference**
on
**Information Systems, Logistics and Supply Chain**
**April 13-16, 2010 - Casablanca, Morocco**

# On Resource Complementarity in Activity Networks

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# Topics

- Introduction and Problem Definition
- Model Description
- Proposed Heuristic
- Conclusions

# Introduction and Problem Definition

- Problem: optimal resource allocation in activity networks under conditions of resource complementarity.
- Complementarity $\Rightarrow$ Enhancement of the efficacy of a "primary" resource (*P*-resource) by adding to it another "supportive" resource (*S*-resource).
- $\uparrow$ Performance $\uparrow$ Quality $\downarrow$ Duration $\Uparrow$ Cost
- How much additional support should be allocated to project activities to achieve improved results most economically?

# Introduction and Problem Definition

- Project in AoA mode of representation: G(N,A)
  - N: set of nodes (events)
  - A: set of arcs (activities)
- Set of primary resources ($P$) with $|P| = \rho$.
- Pool of support resources ($S$), with $|S| = \sigma$.

# Introduction and Problem Definition

- The relevance of each *S*-resources to the *P*-resources may be represented as:

| *S-RESOURCE* → | $S_1$ | … | $S_q$ | … | $S_\sigma$ |
|---|---|---|---|---|---|
| ↓ *P-RESOURCE* | | | | | |
| $r_1$ | $v(1,1)$ | … | $\phi$ | … | $v(1,\sigma)$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $r_p$ | $\phi$ | … | $v(p,q)$ | … | $v(p, \sigma)$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $r_\rho$ | $v(\rho,1)$ | … | $v(\rho,q)$ | … | $\phi$ |

Primary Resource = P with $|P| = \rho$
Supportive Resource = S with $|S| = \sigma$

# Introduction and Problem Definition

- If $0 < v(r_p, s_q) \leq 1$

  It indicates the fraction by which the support resource $s_q$ improves the performance of primary resource $r_p$.
- Typically

  $v(r_p, s_q) \in [0.10, 0.50]$
- Performance of $r_p$ allocated to activity $a$ is augmented to

  $x_{r_p}(a) = x(a, r_p) + v(r_p, s_p)$

**Model (1)**

# Introduction and Problem Definition

- If $1 < v(r_p, s_q) < U < \infty$

  It indicates the multiplier of the *P-resource* allocation.

- Typically

$$1.10 \le v(r_p, s_q) \le 2.0$$

- Performance of $r_p$ allocated to activity $a$ is augmented to

$$x_{r_p}(a) = x(a, r_p) \cdot v(r_p, s_q)$$

**Model (2)**

# Model Description

- Assumption 1: The impact of *S*-resource is additive:
  - Considering a subset $\{s_q\}_{q=1}^{\sigma}$ of the *S*-resources is used in support of *P*-resource $r_p$ in activity *a* then the performance of the former is enhanced to:

$$x_{r_p}(a) = x(a, r_p) + \sum_{q=1}^{\sigma} v(r_p, s_q)$$

  - With $r_p \in P$ allocated to activity, the duration will be $y(a, r_p)$
  - Adding $s_q$ the duration will be denoted by $y_{r_p}(a)$, where:

$$y_{r_p}(a) < y(a, r_p).$$

# Model Description

- The duration of activity *a* using only *P*-resource $r_p$:

$$y(a, r_p) = \frac{w(a, r_p)}{x(a, r_p)}$$

- The duration of activity *a* adding *S*-resource to *P*-resource:

$$y_{r_p}(a) = \frac{w(a, r_p)}{x_{r_p}(a)}$$

$$x_{r_p}(a) = x(a, r_p) + \sum_{q=1}^{\sigma} v(r_p, s_q)$$

# Model Description

- Example:
  - Considering: $w(a, r_p) = 36$ man-days
    $v(r_p, s_q) = 0.50$
    $x(a, r_p) = 1.5$
  - In the absence of the supportive resource the duration of activity would be $y(a, r_p) = 36/1.5 = 24\ days$
  - Considering the supportive resource the newer duration is
    $$y_{r_p}(a) = {}^{36}/_{(1.5 + 0.5)} = 18 \text{ days}$$
  - It means a saving of approximatly **25%.**

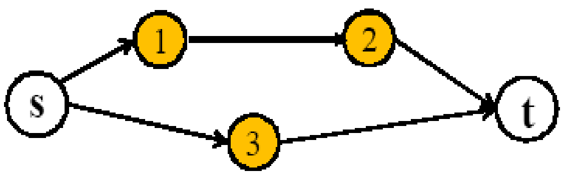
# Model Description

- An activity normally requires the simultaneous utilization of more than one *P*-resource for its execution. The problem then becomes:
  - "At what level should each resource be utilized and which supportive resource(s) should be added to it (if any) in order to optimize a given objective?"
- The processing time of an activity is given by

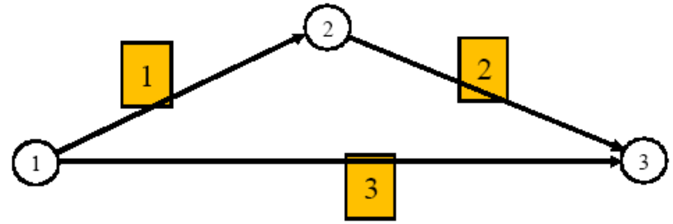
$$y(a) = \underbrace{max}_{all\ r_p} \left\{ y_{r_p}(a) \right\}$$

# Model Description

- Considering the minuscule project below




a) AON representation




b) AOA representation

- Additional Information

| *P-RESOURCE* → | *1* | *2* | *3* | *4* |
|---|---|---|---|---|
| *AVAILABILITY* | 2 | 1 | 3 | 2 |
| *Activity* | | | | |
| *A1* | 16 | 0 | 12 | 12 |
| *A2* | 0 | 7 | 0 | 8 |
| *A3* | 20 | 22 | 0 | 0 |

Work content (in man-days) of the activities.

| | *P-RES* → | *1* | *2* | *3* | *4* |
|---|---|---|---|---|---|
| ↓*S-RES* | ↓ *AVAILABILITY* | | | | |
| *1* | *1* | 0.25 | ϕ | 0.25 | ϕ |
| *2* | *1* | 0.15 | 0.35 | ϕ | ϕ |

The P-S matrix: Impact of *S*-resources on *P*-resources.

# Model Description

- At time 0 we may initiate both activities A1 and A3 because their required *P*-resources are available.
- Assumption 2: Assume for the moment that no support resource is allocated to either activity. Further, suppose that each unit of the primary resource is devoted to its respective activity at level 1; i.e., $x(1, r_1) = 1 = x(1, r_3) = x(1, r_4)$ $x(3, r_1) = 1 = x(3, r_2)$
- The *P*-resource allocation would look as:

| | *P-RESOURCE* | | | |
|---|---|---|---|---|
| *ACTIVITY* | 1 | 2 | 3 | 4 |
| *A1* | 1 | 0 | 1 | 1 |
| *A3* | 1 | 1 | 0 | 0 |
| *TOTAL ALLOCATION* | 2 | 1 | 1 | 1 |

# Model Description

- The duration of the two activities shall be:

| *P-RESOURCE →* | *1* | *2* | *3* | *4* |
|---|---|---|---|---|
| *AVAILABILITY* | 2 | 1 | 3 | 2 |
| *Activity* | | | | |
| *A1* | 16 | 0 | 12 | 12 |
| *A2* | 0 | 7 | 0 | 8 |
| *A3* | 20 | 22 | 0 | 0 |

$$A1: y_1 = max\left\{\frac{16}{1}, \frac{12}{1}, \frac{12}{1}\right\} = 16 \text{ days}$$

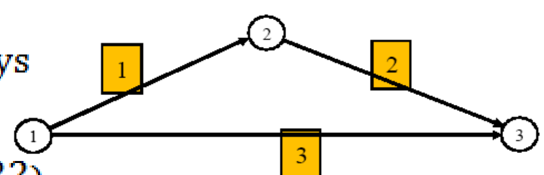
| | *P-RESOURCE* | | | |
|---|---|---|---|---|
| *ACTIVITY* | 1 | 2 | 3 | 4 |
| *A1* | 1 | 0 | 1 | 1 |
| *A3* | 1 | 1 | 0 | 0 |
| *TOTAL ALLOCATION* | 2 | 1 | 1 | 1 |

$$A3: y_3 = max\left\{\frac{20}{1}, \frac{22}{1}\right\} = 22 \text{ days}$$

# Model Description

- At time t = 16 activity A1 completes processing and A2 becomes sequence feasible.
- Unfortunately it cannot be initiated because *P*-resource 2, of which there is only one unit, is committed to A3 which is still on-going. Therefore activity 2 must wait for the completion of A3, which occurs at t = 22.

$$A1: y_1 = max\left\{\frac{16}{1}, \frac{12}{1}, \frac{12}{1}\right\} = 16 \text{ days}$$




$$A3: y_3 = max\left\{\frac{20}{1}, \frac{22}{1}\right\} = 22 \text{ days}$$

# Model Description

- Resource levels for activity 2:

$$x(2, r_2) = 1 = x(2, r_4)$$

- Duration of activity 2:

$$y_2 = max\left\{\frac{7}{1}, \frac{8}{1}\right\} = 8 \text{ days}$$

- Project duration:

$$T^{(1)} = 22 + 8 = 30 \text{ days}$$

- Considering *Ts = 24* days, the project would be *6* days late.

# Model Description

- Impact of the Support Resource
  - Suppose that at the start of the project both support resources were allocated to activity 3 as follows:

$$s_1 \rightarrow r_1 \quad \Longrightarrow \quad x_{r_1}(3) = 1 + 0.25$$

$$s_2 \rightarrow r_2 \quad \Longrightarrow \quad x_{r_2}(3) = 1 + 0.35$$

| | *P-RES* → | *1* | *2* | *3* | *4* |
|---|---|---|---|---|---|
| ↓*S-RES* | ↓ *AVAILABILITY* | | | | |
| *1* | *1* | 0.25 | ϕ | 0.25 | ϕ |
| *2* | *1* | 0.15 | 0.35 | ϕ | ϕ |

# Model Description

- Impact of the Support Resource
  - The duration of the activity 3 would change to:

$$y_3 = max\left\{\frac{20}{1.25}, \frac{22}{1.35}\right\} \approx 16.30 \text{ days}$$

| *P-RESOURCE* → | *1* | *2* | *3* | *4* |
|---|---|---|---|---|
| *AVAILABILITY* | 2 | 1 | 3 | 2 |
| *Activity* | | | | |
| *A1* | 16 | 0 | 12 | 12 |
| *A2* | 0 | 7 | 0 | 8 |
| *A3* | 20 | 22 | 0 | 0 |

# Model Description

- Impact of the Support Resource
  - At t = 16.30 activity 2 can be initiated because primary resource 2 would be freed.
  - If we continue with $x(2, r_2) = 1 = x(2, r_4)$ it will consume the same 8 days to complete and the project duration would be,

$$T^{(2)} = 16.30 + 8 = 24.30$$

  - The project is almost on time.

# Model Description

- Assumption 3: we assume that all costs are linear or piece-wise linear in their argument.
- Model variables:
  - $C^k$ **:** The *k*th uniformly directed cutset *(udc)* of the project network that is traversed by the project progression.
  - $x(a, r_p)$: Level of allocation of (primary) resource $r_p$ to activity *a* (assuming integer values from 1 to $Q_p(p)$ if the activity needs this resource).
  - $x(a, (r_p , s_q))$**:** Level of allocation of secondary resource $s_q$ to primary resource in activity *a* (assuming integer values from 0 to $Q_s\,(q)$).
  - $x_{rp}(a)$ Total allocation of resource $r_p$ (including complementary resource) to activity *a*.
  - $v(r_p , s_q)$**:** Degree of enhancement of *P*-resource $r_p$ by *S*-resource $s_q$.
  - $w(a, r_p)$**:** Work content of activity *a* when *P*-resource $r_p$ is used.

# Model Description

- ***$y_{rp}$(a):*** Duration of activity *a* imposed by primary resource $r_p$ (with or without enhancement from *S*-resource $s_q$).
- ***y(a):*** Duration of activity *a* (considering all resources).
- ***$\rho$:*** Number of primary resources, $\rho = |P|$.
- ***$\sigma$:*** Number of secondary resources, $\sigma = |S|$.
- ***Q(p)(Q(q)):*** Capacity of *P*-resource $r_p$ (*S*-resource $s_q$) available.
- $\gamma_p$ **:** Marginal cost of *P*-resource $r_p$.
- $\gamma_q$ **:** Marginal cost of *S*-resource $s_q$.
- $\gamma_E$ **:** Marginal gain from early completion of the project.
- $\gamma_L$ **:** Marginal loss (penalty) from late completion of the project.
- $t_i$ **:** Time of realization of node *i* (AoA representation), where node 1 is the "start node" of the project and node *n* its "end node".
- $T_s$ **:** Target completion time of the project.

# Model Description

- We refer to an activity as *"a"* and to a node as *i* or *j*.
- The notation $a \equiv (i\ ,\ j)$ means that activity *a* is represented by arc $(i\ ,\ j)$.
- The model functions and constraints will be enumerated next.
- Respect precedence among the activities:

$$t_j \geq t_i + y(a), \ \ \forall a \equiv (i,j) \in A$$

# Model Description

- Define total allocation of resource $r_p$ (including complementary resource) in activity $a$,

$$x_{r_p}(a) = x(a, r_p) + \sum_{s_q} \left( v(r_p, s_q) * x\left(a, (r_p, s_q)\right) \right)$$

# Model Description

- Define the duration of each activity when using each *P*-resource:

$$y_{r_p}(a) = \frac{w(a, r_p)}{x_{r_p}(a)}$$

- Define the activity's duration as the maximum of individual resource durations:

$$y(a) = \max_{all\ r_p} \{y_{r_p}(a)\}$$

# Model Description

- Respect the *P*-resource availability at each *udc* traversed by the project in its execution,

$$\sum_{a \in C^k} x(a, r_p) \leq \mathrm{Q_P(p)}, \qquad \forall p \in P, \forall\, C^k$$

# Model Description

- Difficulties and considerations
  - We do not know a priori the identity of the *udc*'s that shall be traversed during the execution of the project.
  - A circularity of logic is present here: the allocation of the resources is bounded by their availabilities at each *udc*, but these latter cannot be known except after the allocations have been determined.
  - An heuristic approach to this problem will be presented later.

# Model Description

- Respect for the *S*-resources availability for each *udc* traversed by the project in its execution.

$$\sum_{a \in C^k} x\left(a, \left(r_p, s_q\right)\right) \leq Q_S(q) \quad \forall q \in S, \forall C^k$$

# Model Description

- Define earliness and tardiness by:

$$e \geq T_s - t_n$$

$$d \geq t_n - T_s$$

$$e, d \geq 0$$

# Model Description

- The criterion function is composed of two parts:
  - The cost of use of the *P*- and *S*-resources;
  - The gain or loss due to earliness or tardiness, respectively; of the project completion time ($t_n$) relative to its due date.

# Model Description

(i) Cost of resource utilization:

$$c_R(a, r_p) = \left( \gamma_p * \mathrm{x}(\mathrm{a}, r_p) + \gamma_q * \sum_{\mathrm{all}\ \mathrm{s_q}} x\left(a, (r_p, s_q)\right) \right) * w(a, r_p)$$

$$c_R(a) = \sum_{\mathrm{all}\ \mathrm{r_p}} c_R(a, r_p)$$

# Model Description

(ii) Earliness-tardiness costs:

$$c_{ET} = \gamma_E \cdot e + \gamma_L \cdot d$$

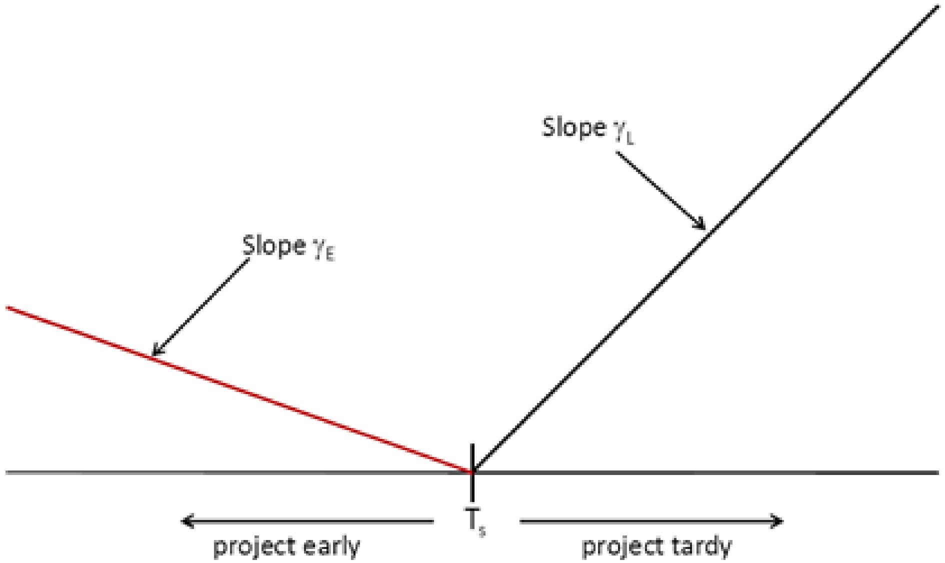



*Figure : Linear cost of earlier and tardier end of Project.*

# Model Description

- The desired objective function may be written simply as,

$$\min z = \sum_{a \in A} c_R(a) + c_{ET}$$

# Proposed Heuristic

- Addressing the problem raised above
  - How can one constrain the aggregate use of the *P*- and the *S*-resources when the identity of the *udc* to which the constraining relation should be applied is known only after the allocations have been made?
  - At the start node 1 the *udc* is known, hence constraints can be imposed.
  - Assume abundant availability of the resources in all subsequent *udc*'s hence these constraints need not be considered.
  - The solution obtained shall identify the next node to be realized the earliest.
  - Repeat the same optimization step at the new node, taking into account the committed resource(s) to the on-going activities from the previous step, assuming abundant availability of the resources in all subsequent *udc*'s. Continue until the project is completed.
  - Observe that the solution obtained is feasible, therefore its value constitutes an upper bound on the optimum cost.

# Conclusions

- The goal of this work was to provide a formal model to some unresolved issues in the management of projects, especially as related to the utilization of supportive resources.
- The relevance of the problem is the opportunity to shape a system that allows not only that we improve the allocation of often scarce resource(s), but also result in reduced uncertainties within the projects, combined with increased performance and lower project costs.


# Conclusions

- There still remains the implementation of the model in an easy-to-use computer code that renders it practically usable.
- This research also unveils several research avenues to be explored. These can be gleaned from the assumptions made. Relaxation of one or more of these assumptions would go a long way towards the resolution of more real life problems.

We thank Prof. Elmaghraby for his contribution in the definition of this problem.

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