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Multiple Resource Allocation Strategies on Projects under Stochastic Conditions

Quantity and Duration approaches

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Introduction: Problem Context

Work Content

$$W_r^a \sim \mathrm{Exp}\left(\lambda_r^a\right)$$

Each resource (r) infers its own work content on an activity (a)

Resource Allocation

$$0 \leqslant l_r^a \leqslant x_r^a \leqslant u_r^a < \infty$$

The resource quantity (x) is constrained on each activity

Introduction: Problem Context

Resource Cost

$$\mathrm{RC}_a = \sum_{r \in \mathrm{R}_a} \left(c_r \times x_r^a \times W_r^a \right)$$

Tardiness Cost

$$\mathrm{TC} = c_L \times \max\left(0, \Upsilon_n - T\right)$$

Total Project Cost

$$C = \mathcal{E}\left[\sum_{a \in \mathrm{A}} \mathrm{RC}_a + \mathrm{TC}\right]$$

Goal: To minimize the total project cost

Introduction: Problem Context

Individual Duration

$$Y_r^a = W_r^a / x_r^a \ , \ r \in \mathrm{R}_a$$

Each resource secures an individual duration

Activity Duration

$$Y_a = \max_{r \in \mathrm{R}_a} \left(Y_r^a \right)$$

The activity duration is the maximum of the individual durations for each of its resources

Introduction: Premise

It makes little sense to expend more of a resource (and incur higher cost) to have the activity duration under this resource shorter than its duration under any other resource.

↓

It is desired to have equal durations (in expectation)

$$\mathcal{E}\left[Y_r^a\right] = \mathcal{E}\left[Y_s^a\right] \; \forall r, s \in \mathrm{R}_a$$

Allocation Strategies

Wish: Avoid wasteful allocations

QORAS

Quantity Oriented Resource Allocation Strategy

DORAS

Duration Oriented Resource Allocation Strategy

Elected Strategy: WBRAS

(Waste Balanced Resource Allocation Strategy)

Allocation Strategies: QORAS

One resource is elected as “pivot” based on which we write equations that describe all the quantities for the other resources, on the same activity, achieving the equality on the individual durations.

$$\mathcal{E}\left[Y_1^a\right] = \mathcal{E}\left[\frac{W_1^a}{x_1^a}\right] = \frac{\mathcal{E}\left[W_1^a\right]}{x_1^a} = \frac{\mathcal{E}\left[W_2^a\right]}{x_2^a} = \mathcal{E}\left[\frac{W_2^a}{x_2^a}\right] = \mathcal{E}\left[Y_2^a\right]$$

⇓

$$x_2^a = \frac{\mathcal{E}[W_2^a]}{\mathcal{E}[W_1^a]} x_1^a$$

Allocation Strategies: QORAS

Example (general configuration)

$$x_1^a \in [0.5,\ 2] \qquad \mathcal{E}\left[W_1^a\right] = 1$$

$$x_2^a \in [1.5,\ 3] \qquad \mathcal{E}\left[W_2^a\right] = 2$$

Decision Equation

$$x_2^a = 2 \times x_1^a$$

Allocation Strategies: QORAS

Example (A)

$$x_1^a = 1.5$$

Decision

$$x_2^a = 3$$

From a starting value of pivot resource (1), a valid quantity for resource 2 is evaluated

Allocation Strategies: QORAS

Example (B)

$$x_1^a = 0.5 \xrightarrow{\text{Decision}} x_2^a = 1$$

Problem: The result is not within bounds

Workaround: Truncation within legal bounds

$$x_2^a = 1 \xrightarrow{\text{Truncation}} \hat{x}_2^a = 1.5$$

Problem: The relation defined by the decision is no longer valid

Workaround: Back trace the corresponding value for the pivot

Allocation Strategies: QORAS

Example (B – continued)

$$\hat{x}_2^a = 1.5$$

Restoration →

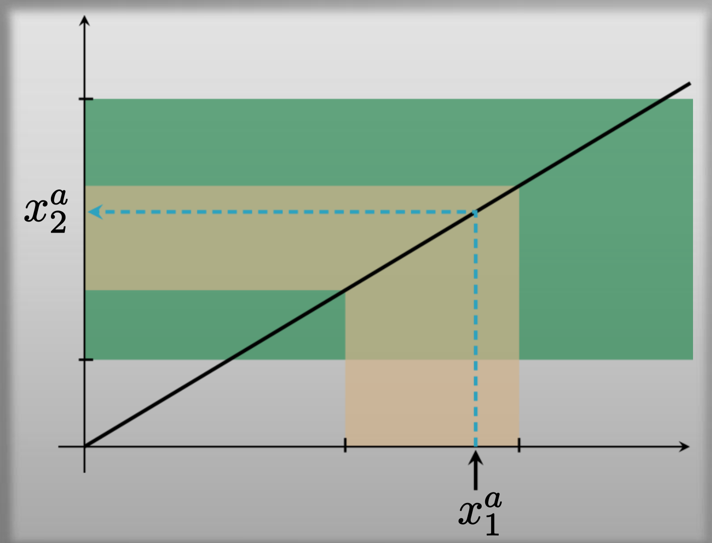
$$\hat{x}_1^a = 0.75$$

From the initial value of **0.5** for the pivot, we evaluate the valid allocation vector **(0.75, 1.5)** after successful truncation and restoration along the process

Allocation Strategies: QORAS

General Scenario A

Range of the decision function is included by the allocation interval of the non-pivotal resource



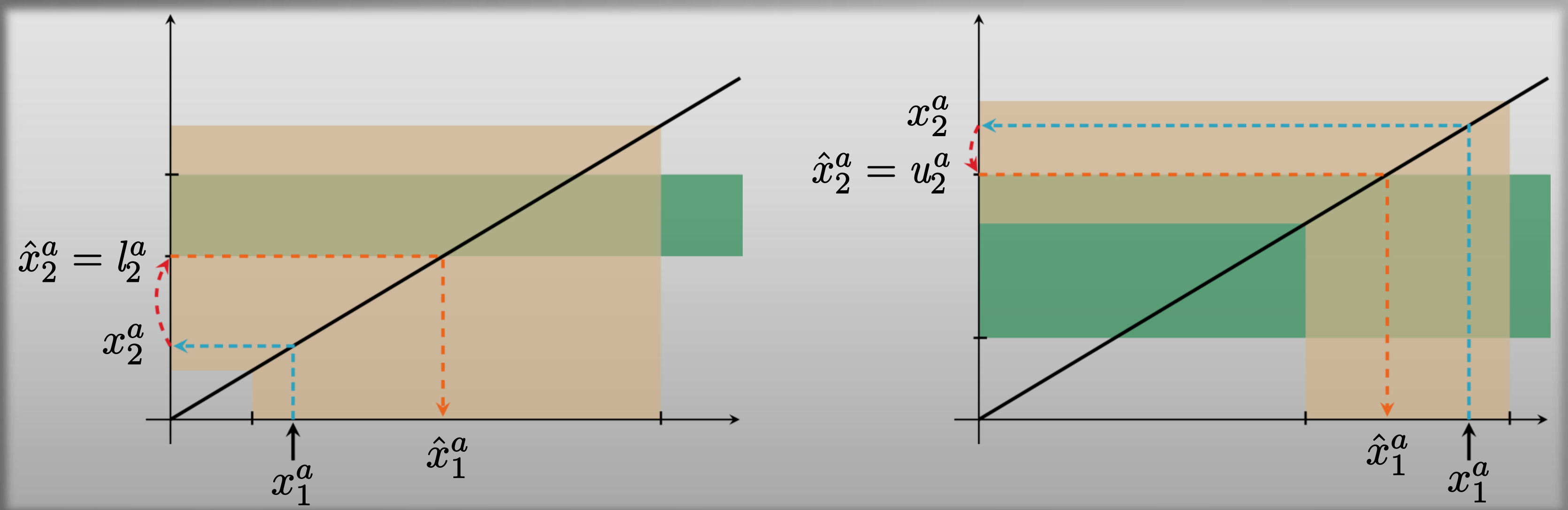
The decision over the pivot resource always suffice

Decision

Allocation Strategies: QORAS

General Scenario B

Range of the decision function is not fully included by the allocation interval of the non-pivotal resource



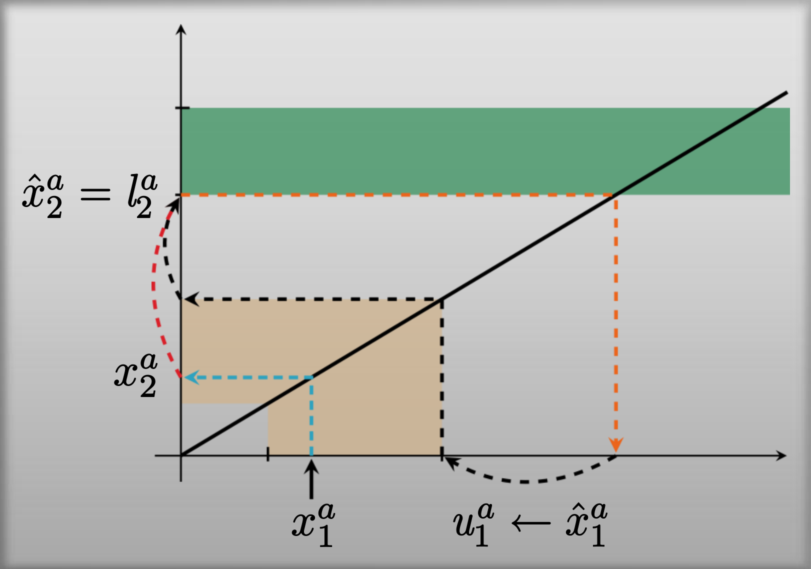
The decision over the pivot resource not always suffice. There are regions where truncation followed by restoration is required

Decision | Truncation | Restoration

Allocation Strategies: QORAS

General Scenario C

Range of the decision function <u>does not intersect</u> the allocation interval of the non-pivotal resource



It is impossible to have equal individual durations

Decision | Truncation | Restoration

Allocation Strategies: QORAS

General Process (2 resources)

x_1^a → x_2^a → $\hat{x}_2^a$ → $\hat{x}_1^a$

Allocation Vector
(before correction)

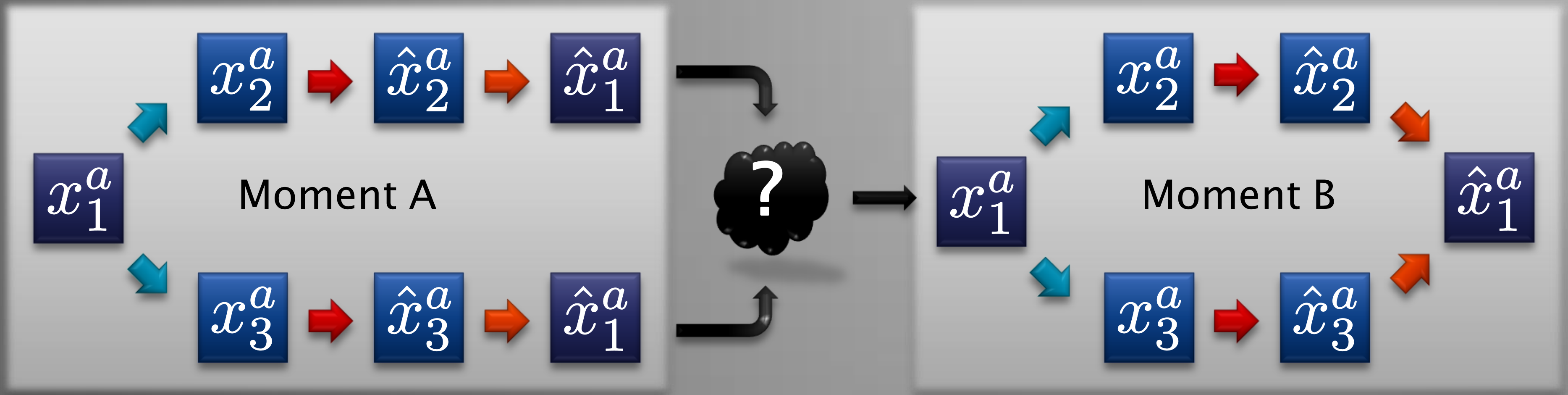
Allocation Vector
(after correction)

Apart from the case where the equality is impossible, for each quantity of the pivot resource it is possible to evaluate a suitable allocation vector (eventually after correction)

Decision | Truncation | Restoration

Allocation Strategies: QORAS

General Process (3 resources)



With 3 resources, the initial moment may lead to divergent values for the pivot resource. Thus, it is required a "balancing" stage where a suitable pivot value must be evaluated to avoid that in the second moment

Balancing Decision Truncation Restoration

Allocation Strategies: QORAS

Conclusion

Positive:

- Intuitive and straightforward
- Based on manipulation of quite simple equations

Negative:

- Deals rather roughly with the case where it is impossible to achieve equal individual durations
- Further difficult to generalize for an arbitrary number of resources due to the necessity of establishing suitable pivotal quantities

Allocation Strategies: DORAS

Establishes all the possible equal individual durations (secured by each resource in the same activity). Then, this set is used to determine the allocation vectors securing those durations.

Instead of using only the expected value of the work content (like QORAS), it uses all the combinations of a set of realizations of the work content of each resource involved.

Possible durations of resource $r \in \mathrm{R}_a$

$$y_r^a = \left[\frac{\omega_r^a}{u_r^a}, \frac{\omega_r^a}{l_r^a}\right], \ \omega_r^a \text{ a realization of } W_r^a, \ l_r^a \leqslant x_r^a \leqslant u_r^a$$

Allocation Strategies: DORAS

Example (general configuration)

Allocation Domain

$$x_1^a, x_2^a, x_3^a \in [0.5, 1.5]$$

Work Content

$$W_1^a \sim \mathrm{Exp}\,(0.1) \qquad W_2^a \sim \mathrm{Exp}\,(0.2)$$

$$W_3^a \sim \mathrm{Exp}\,(0.01)$$

Allocation Strategies: DORAS

Example (Work content realization)

$$\mathcal{W}_1^a = \{1.37; 4.77; 10; 23.86\}$$

$$\mathcal{W}_2^a = \{0.68; 2.38; 5; 11.93\}$$

$$\mathcal{W}_3^a = \{13.7; 47.68; 100; 238.63\}$$

Each set is constructed based on the quartiles of each distribution. Each value on the set is the mean on each of 4 domains delimited by the quartiles.

Allocation Strategies: DORAS

Example (A)

$$\omega_1^a = 10 \Rightarrow y_1^a = [6.67, 20]$$

$$\omega_2^a = 5 \Rightarrow y_2^a = [3.33, 10]$$

$$\omega_3^a = 13.7 \Rightarrow y_3^a = [9.13, 27.4]$$

For each realization on each resource it is easy to determine the individual duration range

Allocation Strategies: DORAS

Example (A – continued)

$$\mathcal{Y}_1^a = [6.67, 20] \quad \mathcal{Y}_2^a = [3.33, 10] \quad \mathcal{Y}_3^a = [9.13, 27.4]$$

$$\mathcal{Y}_a = [9.13, 10]$$

Equal individual durations interval secured by all the resources
(by simple interval intersection)

Allocation Strategies: DORAS

Example (A – continued)

$$\psi \in \mathcal{Y}_a = [9.13, 10]$$

$$x_1^a = 10/\psi$$

$$x_2^a = 5/\psi \quad \xrightarrow{\psi=10} \quad \mathrm{X}_a = (1, 0.5, 1.37)$$

$$x_3^a = 13.7/\psi$$

Each (equal) duration leads to an allocation vector

Allocation Strategies: DORAS

Example (B)

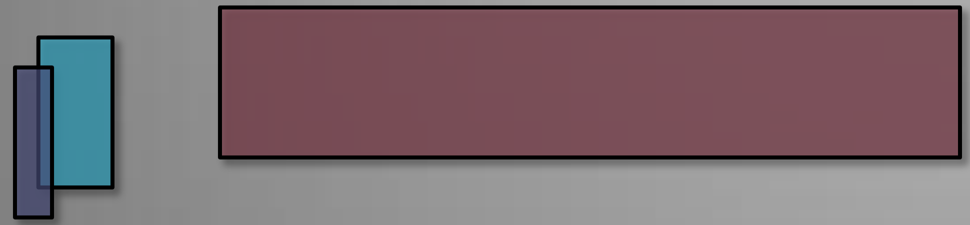
$$\omega_1^a = 1.37 \Rightarrow y_1^a = [0.91, 2.74]$$
$$\omega_2^a = 0.68 \Rightarrow y_2^a = [0.45, 1.36]$$
$$\omega_3^a = 13.7 \Rightarrow y_3^a = [9.13, 27.4]$$

For different realizations, we have different duration intervals

Allocation Strategies: DORAS

Example (B – continued)

$\mathcal{Y}_1^a = [0.91, 2.74]$ $\mathcal{Y}_2^a = [0.45, 1.36]$ $\mathcal{Y}_3^a = [9.13, 27.4]$



Problem: The intersection of all intervals is <u>null</u>
Resource 3 alone is securing the longer durations
Resources 1 and 2 can secure equal durations themselves

Allocation Strategies: DORAS

Example (B – continued)

A single resource secures all the longer durations

↓

The longest duration (equal or not) secured by the other two resources themselves is shorter than the shortest secured above

↓ ↓

Resource 3 alone secures the activity duration

Less quantity implies less cost

↓

The resource securing the longer durations has its allocations made accordingly with that interval. All others are allocated with their smallest permissible amount

Allocation Strategies: DORAS

Example (B – continued)

$$\psi \in \mathcal{Y}_a = [9.13, 27.4]$$

$$x_1^a = 0.5$$

$$x_2^a = 0.5 \quad \xrightarrow{\psi=10} \quad \mathrm{X}_a = (0.5, 0.5, 1.37)$$

$$x_3^a = 13.7/\psi$$

Allocating the minimum for resources 1 and 2.
Resource 3 allocated accordingly to the duration interval

Allocation Strategies: DORAS

Example (C)

$$\omega_1^a = 4.77 \Rightarrow y_1^a = [3.18, 9.54]$$

$$\omega_2^a = 2.38 \Rightarrow y_2^a = [1.59, 4.77]$$

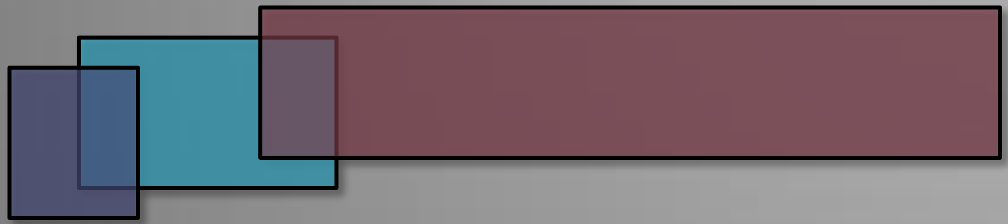
$$\omega_3^a = 13.7 \Rightarrow y_3^a = [9.13, 27.4]$$

Again, we start by evaluating the duration intervals

Allocation Strategies: DORAS

Example (C – continued)

$\mathcal{Y}_1^a = [3.18, 9.54]$ $\mathcal{Y}_2^a = [1.59, 4.77]$ $\mathcal{Y}_3^a = [9.13, 27.4]$



Problem: The intersection of all intervals is <u>null</u>
Resource 1 secures equal durations with each one of the other resources.

Allocation Strategies: DORAS

Example (C – continued)

Two resources (1 and 3) secure all the (equal) longer durations

↓

The resource 1 also secures equal (but shorter) durations with resource 2

↓ ↓

Resource 2 does not affect the activity duration

Less quantity implies less cost

↓

The resources securing the longer equal durations, either exclusively or not, are allocated accordingly to that interval. All others are allocated with their smallest permissible amount

Allocation Strategies: DORAS

Example (C – continued)

$$\psi \in \mathcal{Y}_a = [9.13, 9.54]$$

$$x_1^a = 4.77/\psi$$
$$x_2^a = 0.5 \qquad \xrightarrow{\psi=9.3} \mathrm{X}_a = (0.51, 0.5, 1.47)$$
$$x_3^a = 13.7/\psi$$

Allocating the minimum for resource 2.
Resources 1 and 3 allocated accordingly to the duration interval

Allocation Strategies: DORAS

General Process

For each combination of work content realizations

Evaluate all the individual durations

select the non-null intersection yielding the greatest equal durations – critical class

Those resources securing the critical class are allocated accordingly to these durations.

All others, are allocated with their smallest permissible quantity amounts.

Allocation Strategies: DORAS

Conclusion

Positive:

- The process is easy to apply to any number of resources
- Copes with the case where equal durations (with all resources) are impossible in a simple way

Negative:

- Extremely complex (exponential on the number of resources and work content realizations)
- On the critical class, the securing resources do not contribute with the same amplitude. Thus, a suitable "discretization" of the duration interval, for computational purposes, may prove difficult

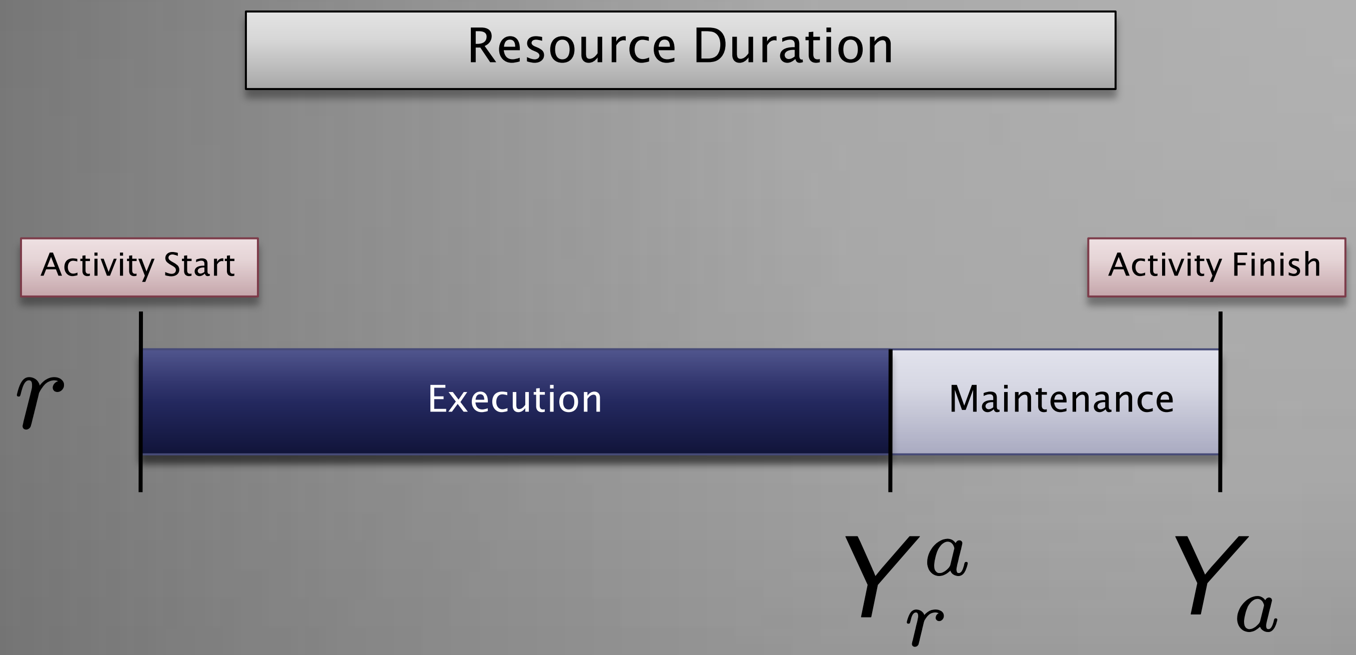
Allocation Strategies: WBRAS

Instead of actively ensure equality among the individual durations, it introduces a penalty cost (called maintenance cost) to the resources with shorter duration

The maintenance cost is related to the resource idle time (called waste time)

Conceptually, with this strategy all the individual durations are composite and equal, on each activity.

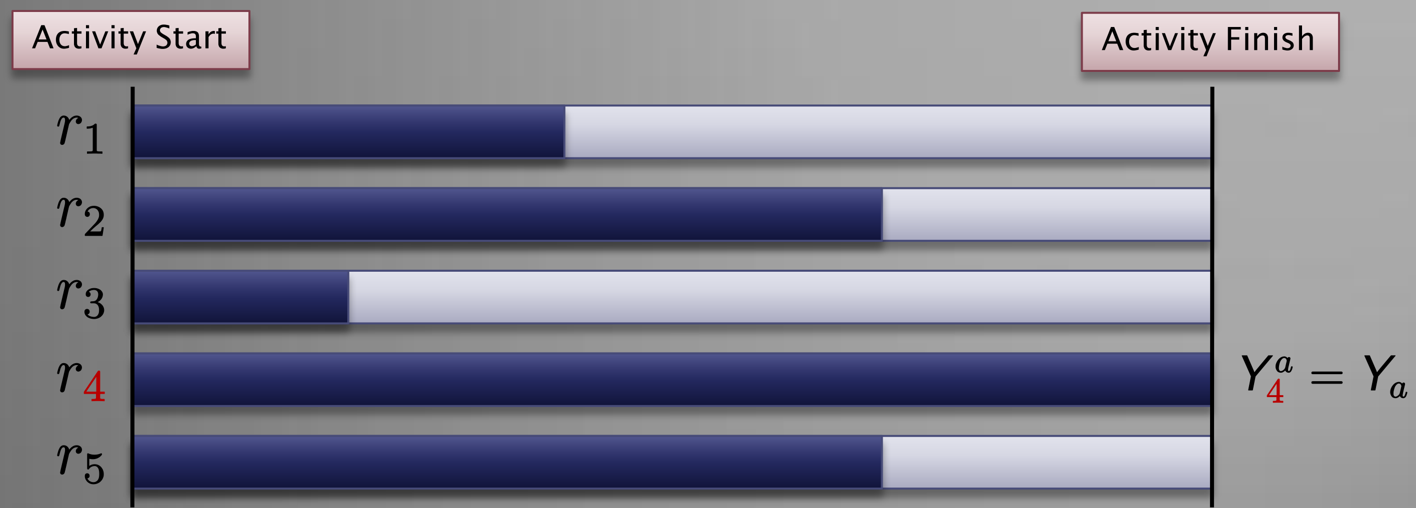
Allocation Strategies: WBRAS



Each resource individual duration is the sum of the duration of the execution/active phase with the maintenance/idle phase

Allocation Strategies: WBRAS

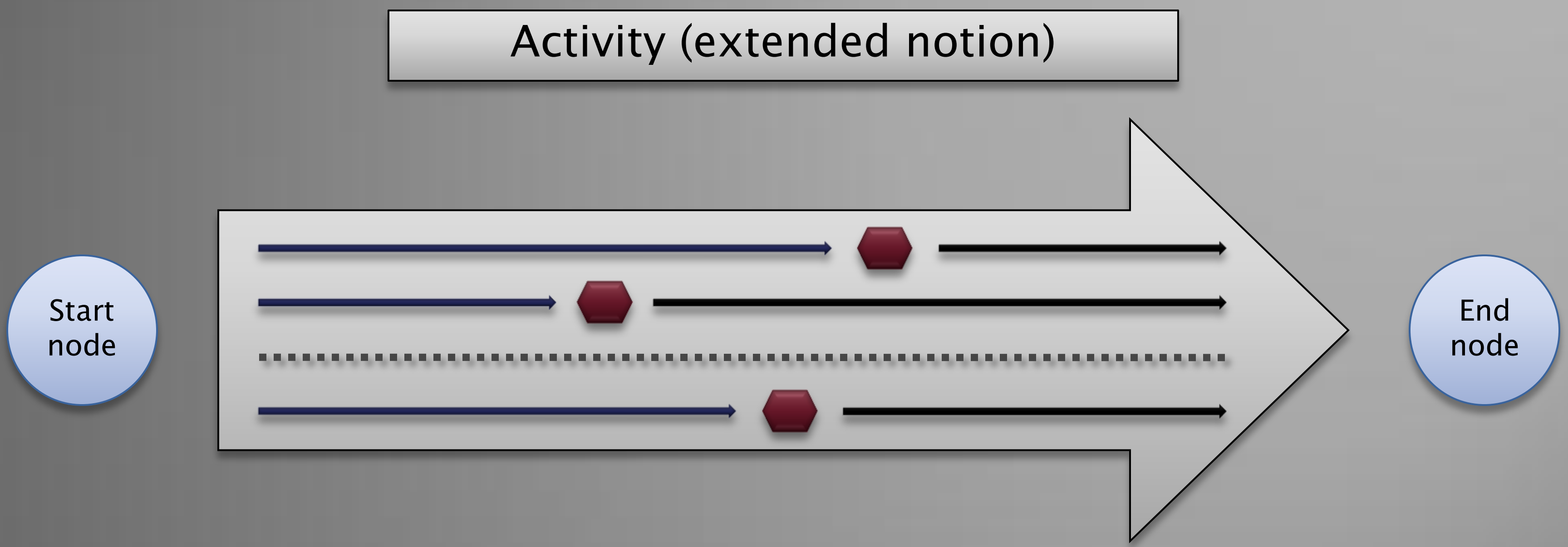
Duration and Cost



The activity duration is equal to the longer execution duration of its resources

To all other resources, a maintenance fee is applied to the idle duration (maintenance cost)

Allocation Strategies: WBRAS



We may conceptualize an activity as being a set of sub-activities in parallel, one per each resource, and composed of an execution phase followed by a maintenance phase

Resource Active Phase · Maintenance Start · Resource Idle Phase

Allocation Strategies: WBRAS

New Assumption

New Factor: Maintenance

↓

Project Cost is now a sum not only of quantity cost and tardiness cost, but also maintenance cost

↓

New Assumption: WBRAS must be applied to those projects being explicitly "maintenance aware"

Allocation Strategies: WBRAS

Conclusion

Positive:

- The process is easy to apply to any number of resources
- Simple and intuitive
- Does not struggle with the impossibility of equal durations
- The penalty may be interpreted has a measure for the maintenance of idle resources (e.g. storage) on practical “real-life” projects

Negative:

- The new (extra) factor is injected on the total project cost

Allocation Strategies: Review

QORAS	DORAS	WBRAS
• Intuitive • Hard to generalize	• Easy to generalize • Exponential complexity	• Straightforward • Introduces a new factor in the cost optimization

We have only further develop the WBRAS for implementation.

The study of QORAS and DORAS was discontinued.
But, each one gives its own insight to the multiple resource allocation problematic.

Future developments on this area, should start by observing their positive and negative points

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