Experimental Results of an Adaptive Resource Allocation Technique to Stochastic Multimodal Projects

Anabela P. Tereso (UM - Portugal)
M. Madalena T. Araújo (UM - Portugal)
Salah E. Elmaghraby (NCSU - USA)

Topics

1 - Problem Definition

2 - The DP Model

3 - Example Network

4 - Results

5 - Application Development

6 - Tested Networks

7- Complexity of the DP Model

1. Problem Definition (1/4)

- Given a multimodal activity network under stochastic conditions, we want to optimize the resource allocation to minimize cost

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Optimization via DP

1. Problem Definition (2/4)

- Network Representation
  - Activity-on-arc
- Activity (A - set of activities)
  - Multimodal
- Work Content
  - Random $\rightarrow$ $\boxed{W_a \sim \exp(\lambda_a)}$
- Resource Allocation
  - Lower and upper bounds on allocations

$\rightarrow$ $\boxed{0 \le l_a \le x_a \le u_a}$

1. Problem Definition (3/4)

- Duration
  - $Y_a = W_a / X_a$
- Resource Cost

Assumed quadratic in allocation for the duration

  - $RC_a \propto x_a^2\, Y_a = x_a\, W_a$
- Due date
  - T
- Tardiness Cost
  - $TC = c_L \max\{0, \Upsilon_n\text{-}T\}$

1. Problem Definition (4/4)

- Goal
  - Determine the resource allocation vector X_a, such that the total expected cost is minimized

$$\boxed{\min_{x} E\left\{ \sum_{a \in A} x_a . W_a + c_L . \max\{0, \Upsilon_n - T\} \right\}}$$

2. The DP Model (1/8)

- D - subset of decision variables
- F- subset of activities to be ‘conditioned upon’

$\downarrow$

each ‘udc’ of the network contains exactly one decision variable

2. The DP Model (2/8)

- Stage of DP
  - epoch of decision on $x_a \in D$
- At each stage
  - optimization over one decision variable
- Nº of stages
  - $K = | D | = | A | - | F |$
- State
  - $s_k = (t_{i_1}, \ldots, t_{i_r})$

2. The DP Model (3/8)

- Stage "reward"
  - For last stage
    - resource cost + tardiness cost
  - For other stages
    - resource cost
- Stage numbering
  - Backwards
    - Stage K: K stages to go to complete the project

2. The DP Model (4/8)

- DP transformation function (for stages 2..K)

$$f_k(s_k|F) = \min_{x_k \in D} E\{ x_k.W_k + E\, f_{k-1}(s_{k-1}|F)\} \quad (1)$$

- Deconditioning

$$f(s_k=0) = \min_{F} f_k(s_k|F)$$

- Solution via DP
  - policy that prescribes the optimal resource allocation under every conceivable state of the project as it progresses over time

2. The DP Model (5/8)

- Application of the DP model
  - Process used to select set F
    1. Determine the longest path in the network
    2. The activities on the longest path will be decision variables (set D)
    3. The others will be the activities to be fixed (set F)
  - Resource cost of fixed variables

$$\boxed{rcf = E \sum_{i \in F} x_i . W_i = \sum_{i \in F} x_i . E(W_i)}$$

2. The DP Model (6/8)

– First stage

$$f_1(s_1|F) = \min_{x_{[1]} \in D} E\,\{ x_{[1]}.W_{[1]} + rcf + c_L.\,E\,(U) \}$$

where $U = \max\{ 0 , \Upsilon_n\text{-}T \}$

– Next stages

$$f_k(s_k|F) = \min_{x_k \in D} E\,\{ x_k.W_k + E\, f_{k-1}(s_{k-1}|F)\}$$

2. The DP Model (7/8)

- We have: best allocation conditioned on F
- Repeat procedure for all possible fixed allocations to the activities in the set F

$$\downarrow$$

- Optimal resource allocation to the activities emanating from node 1

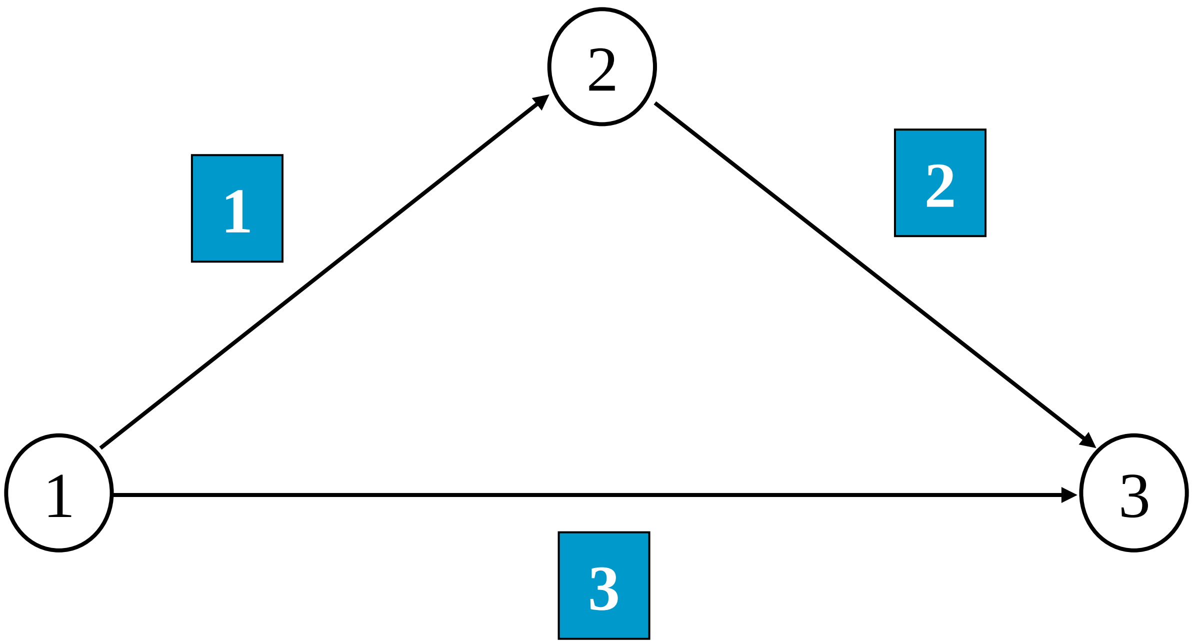
2. The DP Model (8/8)

– Policy afterwards: depends on the state of the process

$$\downarrow$$

– Adaptive nature of the DP approach: later allocations must await the realization of preceding activities as the project evolves over time

3. Example Network (1/5)



3. Example Network (2/5)

Activity a	Parameter λ_a	Expected Work Content
1	0.20	5.00
2	0.10	10.00
3	0.07	14.29

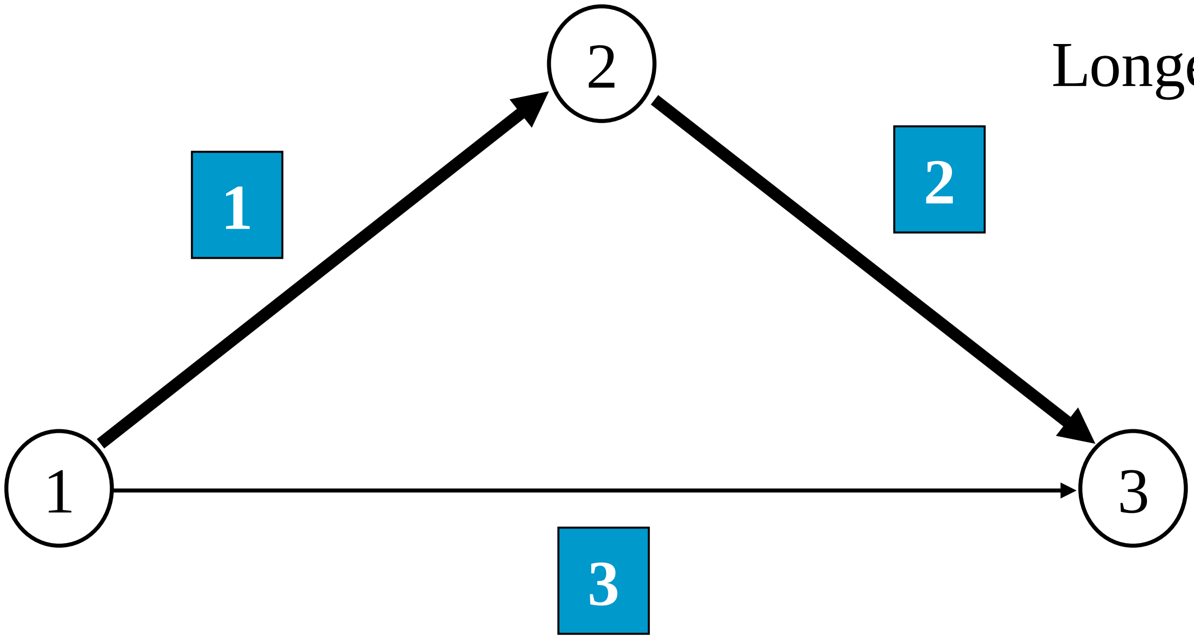
$T=65$

$C_L=5$

$0.5 \leq x_a \leq 1.5$

Unit: weeks

3. Example Network (3/5)



Longest Path

$D = \{x1, x2\}$ $F=\{\widehat{x3}\}$

3. Example Network (4/5)

- x discretized: {0.5, 0.75, 1, 1.25, 1.5}
- W: discretized in 4 values

  p.e. W2 ~ exp(0.1) $\rightarrow$ {1.44, 4.9, 10.4, 19.83}

- rcf = x_3 . E (W_3) = x_3 / 0.07

3. Example Network (5/5)

- DP iterations
  - Stage 1

$$f_1(\,t_2 \mid F=\{3\}) = rcf + \min_{x2} E\,\{\, x_2.W_2 + 2.\,E\,(U)\,\}$$

$$U = \max\{\,0\,,\,\Upsilon_3 - T\,\}$$

$$\Upsilon_3 = \max\{\,t_2 + W_2/x_2,\; W_3/x_3\,\}$$

  - Stage 2

$$f_2(\,t_1=0 \mid F=\{3\}) = \min_{x1} E\,\{\, x_1.W_1 + E\,[f_2\,(\Upsilon_2)]\,\}$$

$$\Upsilon_2 = W_1/x_1$$

4. Results (1/1)

- Computational Tests
  - Pentium III, 650 MHz, 128 MB
  - Windows Millennium

- Solution

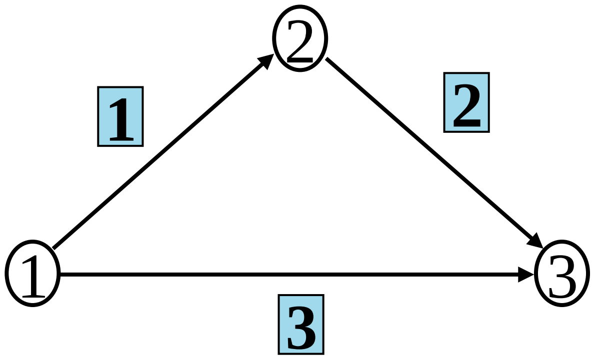
  $(x_1^*, x_3^*) = (1.0, 1.0)$

  at a total expected cost of 45.53

  (running time: 4.8 s)

5. Application Development (1/7)

- Implementation Language
  - MatLab
- Data Structure



Net

Activity	1	2	3
Origin Node	1	2	1
Terminal Node	2	3	3
λ	0.2	0.1	0.07
Lower bound	0.5	0.5	0.5
Upper bound	1.5	1.5	1.5

5. Application Development (2/7)

- Input Parameters
  - n
  - Net
  - C_L
  - T

5. Application Development (3/7)

- The Stages of the Dynamic Program

$$\equiv$$

number of decision variables

$$\Downarrow$$

longest path $\longrightarrow$ D

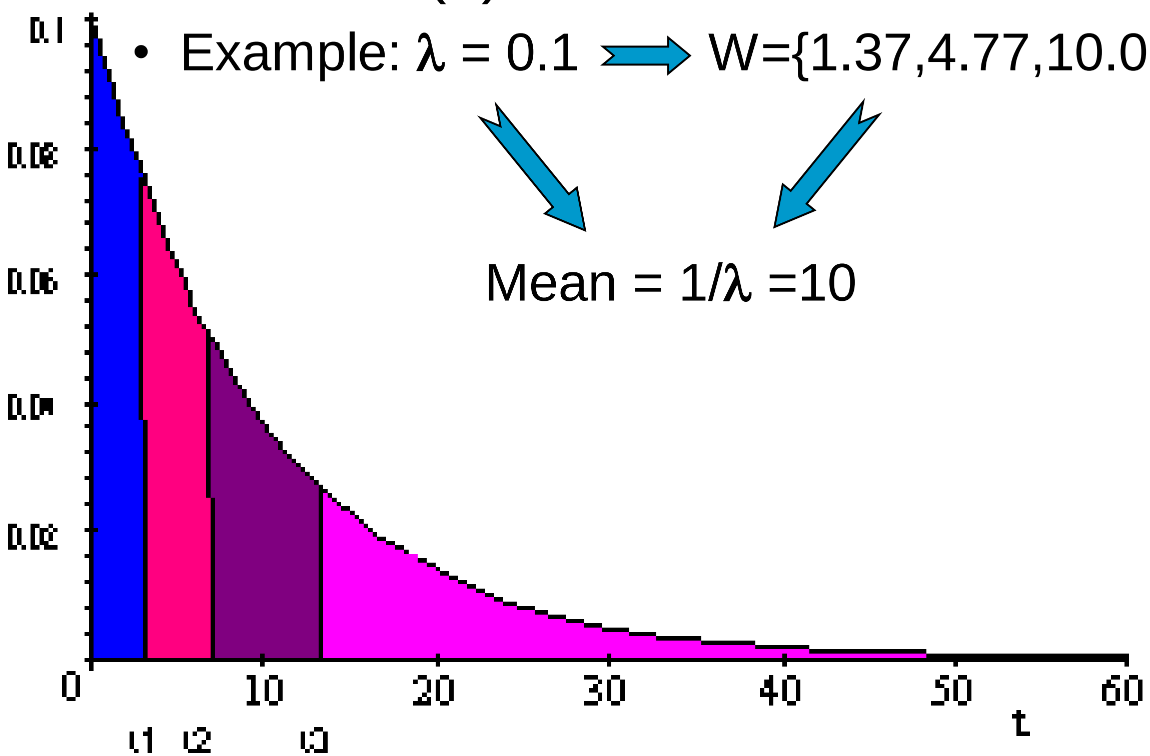
others $\longrightarrow$ F

5. Application Development (4/7)

- Procedures developed
  - *DecisionVars* D={1,2}
  - *SumNodesLen* NL={$\overset{1}{0}$,$\overset{2}{1}$,$\overset{3}{2}$}
  - *NodesLP* NLP={1,2,3}
  - *ActivitiesLP* ALP={1,2}

5. Application Development (5/7)

- The discretization of the Work Content
  - *GenerateW*(λ)



    - Example: $\lambda = 0.1$ → W={1.37,4.77,10.00,23.86}

Mean = $1/\lambda$ =10

$Y_{\min} = \frac{w_{\min}}{x_{\max}}$,

5. Application Development (6/7)

- Bounds on Node Realization Times
  - *Durations*
  - *GenerateTlimits*
    - Node 1: Tlim=0
    - Other nodes:

$$Y_{\min} = \frac{w_{\min}}{x_{\max}} \qquad Y_{\max} = \frac{w_{\max}}{x_{\min}}$$

$$\boxed{T_{\lim j}(1) = \max_{i \prec j}\{T_{\lim i}(1) + Y(i,j)_{\min}\}; \qquad j = 2,\ldots,m.}$$

$$\boxed{T_{\lim j}(2) = \max_{i \prec j}\{T_{\lim i}(2) + Y(i,j)_{\max}\}; \qquad j = 2,\ldots,m.}$$

  - *GenerateTvalues* Tval.T / Tval.step

5. Application Development (7/7)

- Code for the DP Iterations

Network topology

Compose the code dynamically

- *generateMainCode*
- *generateDps1Code*
- *generateDpsNCode*

Matlab: reash

6. Tested Networks

- Projects tested: From 5 to 18 activities
- Internet address:

  www.eng.uminho.pt/~dps/anabela

  (Topic: research)
- E-mail address:

  anabelat@dps.uminho.pt

7. Complexity of the DP Model

$$\boxed{O(N^2 . k_1^N . k_2^{N^2})}$$

- N: number of nodes
- k_1: number of discrete points of the time of realization of the nodes
- k_2: possible allocation of resources at each state

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