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On Resource Complementarity in Activity Networks – Further Results

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Topics

- Introduction and Problem Definition
- Model Description
- Description of the Procedure Adopted
- Results
- Conclusions

Introduction and Problem Definition

- This work is concerned with optimal resource allocation in activity networks under conditions of resource complementarity.
- Complementarity can be incorporated by the enhancement of the efficacy of a “primary” resource by adding to it another “supportive” resource.
- ↑Performance ↑Quality ↓Duration ⇧ Cost
- *How much additional support should be allocated to project activities to achieve improved results most economically?*

Introduction and Problem Definition

- Project in AoA mode of representation: G(N,A)
  - N: set of nodes (events)
  - A: set of arcs (activities)

- There is a set of "primary" resources (P) with $|P| = \rho$.
- There is a pool of "support" resources (S), with $|S| = \sigma$.

Introduction and Problem Definition

- The number of support resources varies with the activity and the primary resource.
- The applicability and impact of support resources are presented below.

$\downarrow$*P-Res/S-Res* $\rightarrow$	s_1	$\cdots$	s_q	$\cdots$	s_σ
r_1	$v(1,1)$	$\cdots$	$\emptyset$	$\cdots$	$v(1,\sigma)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
r_p	$\emptyset$	$\cdots$	$v(p,q)$	$\cdots$	$v(p,\sigma)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
r_ρ	$v(\rho,1)$	$\cdots$	$v(\rho,q)$	$\cdots$	$\emptyset$

Primary Resource = P with $|P| = \rho$
Supportive Resource = S with $|S| = \sigma$

Model Description

- The impact of *S*-resource is additive:

$$x_a\left(r_p, \{s_q\}_{q=1}^{\sigma}\right) = x_a(r_p) + \sum_{q=1}^{\sigma} v(r_p, s_q)$$

  - With $r_p \in P$ allocated to activity, the duration will be $y_a(r_p)$
  - Adding s_q the duration will be denoted by $y_a(r_p, s_q)$, where:

$$y_a(r_p, s_q) < y_a(r_p)$$

Model Description

- The duration of activity *a* using only *P*-resource r_p:

$$y_a(r_p) = \frac{w_a(r_p)}{x_a(r_p)}$$

- The duration of activity *a* adding *S*-resource to *P*-resource:

$$y_a(r_p, s_q) = \frac{w_a(r_p)}{x_a(r_p, s_q)}$$

- The processing time of an activity is given by

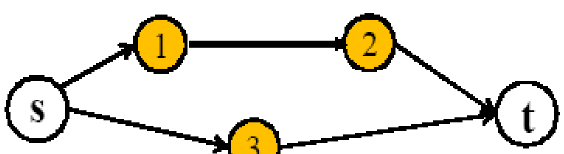
$$y(a) = \underbrace{max}_{all\ r_p} \{y_a(r_p)\}$$

Model Description

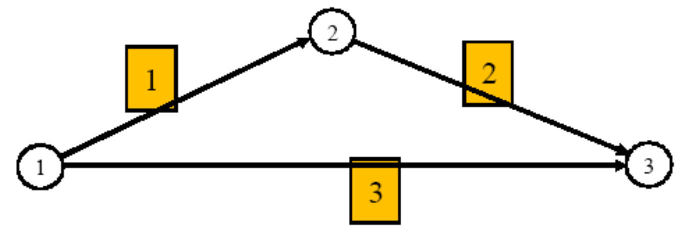
- Example 1:
  - Considering: $w(a, r_p) = 36$ man-days
    $v(r_p, s_q) = 0.50$
    $x(a, r_p) = 0.85$
  - In the absence of the supportive resource the duration of activity would be
    $$y(a, r_p) = 36/0.85 = 42.35 \text{ days.}$$
  - Considering the supportive resource the newer duration is
    $$y_{r_p}(a) = 36 \,/\, ((0.85 + 0.5) = 26.67 \text{ days.}$$
  - It means a saving of approximatly **37%.**

Model Description

- Considering the minuscule project below



a) AON representation



b) AOA representation

- Additional Information

P-RESOURCE →	*1*	*2*	*3*	*4*
AVAILABILITY	2	1	3	2
Activity				
A1	16	0	12	12
A2	0	7	0	8
A3	20	22	0	0

Work content (in man-days) of the activities.

	P-Resource →	*1*	*2*	*3*	*4*
↓*S-Resource*	↓ *Availability*				
1	*1*	0.25	ϕ	0.25	ϕ
2	*2*	0.15	0.35	ϕ	ϕ

The P-S matrix: Impact of *S*-resources on *P*-resources.

Model Description

- The duration of the two activities shall be:

P-RESOURCE →	*1*	*2*	*3*	*4*
AVAILABILITY	2	1	3	2
Activity				
A1	16	0	12	12
A2	0	7	0	8
A3	20	22	0	0

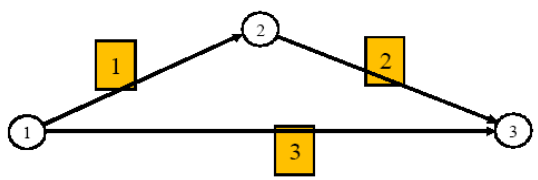
$$A1: y_1 = max\left\{\frac{16}{1}, \frac{12}{1}, \frac{12}{1}\right\} = 16 \text{ days}$$

	P-RESOURCE			
ACTIVITY	1	2	3	4
A1	1	0	1	1
A3	1	1	0	0
TOTAL ALLOCATION	2	1	1	1

$$A3: y_3 = max\left\{\frac{20}{1}, \frac{22}{1}\right\} = 22 \text{ days}$$

Model Description

- At time $t = 16$ activity A1 completes processing and A2 becomes sequence feasible.



$$A1: y_1 = max\left\{\frac{16}{1}, \frac{12}{1}, \frac{12}{1}\right\} = 16 \text{ days}$$

$$A3: y_3 = max\left\{\frac{20}{1}, \frac{22}{1}\right\} = 22 \text{ days}$$

P-RESOURCE →	*1*	*2*	*3*	*4*
AVAILABILITY	2	1	3	2
Activity				
A1	16	0	12	12
A2	0	7	0	8
A3	20	22	0	0

Model Description

- Resource levels for activity 2:

$$x(2, r_2) = 1 = x(2, r_4)$$

- Duration of activity 2:

$$y_2 = max\left\{\frac{7}{1}, \frac{8}{1}\right\} = 8 \text{ days}$$

- Project duration:

$$t_3 = 22 + 8 = 30 \text{ days}$$

- Considering *Ts = 24* days, the project would be *6* days late.

Model Description

- Impacts of the Support Resource
  - Suppose that at the start of the project both support resources were allocated to activity 3 as follows:

$$s_1 \rightarrow r_1 \quad \Rightarrow \quad x_{r_1}(3) = 1 + 0.25$$

$$s_2 \rightarrow r_2 \quad \Rightarrow \quad x_{r_2}(3) = 1 + 0.35$$

	P-Resource →	1	2	3	4
↓S-Resource	↓ Availability				
1	1	0.25	ϕ	0.25	ϕ
2	2	0.15	0.35	ϕ	ϕ

$$y_3 = max\left\{\frac{20}{1.25}, \frac{22}{1.35}\right\} \approx 16.30 \text{ days} \quad \Rightarrow \quad t_3 = 16.30 + 8 = 24.30$$

Model Description

- Assuming that all costs are linear or piece-wise linear in their argument.
  - Let:
    - C^k:The *k*th uniformly directed cutset *(udc)* of the project network that is traversed by the project progression.$k=1, 2, \ldots, K$.
    - $x_a(r_p)$:Level of allocation of (primary) resource r_p to activity *a* (assuming integer values from 1 to $Q_p(p)$ if the activity needs this resource).
    - $x_a^{r_p}(s_q)$: Level of allocation of secondary resource s_q to primary resource r_p in activity *a* (assuming integer values from 0 to $Q_s(q)$).
    - $x_a\left(r_p, \{s_q\}_{q=1}^{\sigma}\right)$:Total allocation of resource r_p (including complementary resource) to activity *a*.
    - $v(r_p, s_q)$:Degree of enhancement of *P*-resource r_p by *S*-resource s_q.
    - $w_a(r_p)$:Work content of activity *a* when *P*-resource r_p is used.

Model Description

- $y_a(r_p, \{s_q\}_{q=1}^{\sigma})$:Duration of activity *a* imposed by primary resource r_p (with or without enhancement from *S*-resource s_q).
- $y(a)$:Duration of activity *a* (considering all resources).
- ρ :Number of primary resources, $\rho = |P|$.
- σ :Number of secondary resources, $\sigma = |S|$.
- $Q_P(p)(Q_S(q))$:Capacity of *P*-resource r_p (*S*-resource s_q) available.
- γ_p :Marginal cost of *P*-resource r_p.
- γ_q :Marginal cost of *S*-resource s_q.
- γ_E :Marginal gain from early completion of the project.
- γ_L :Marginal loss (penalty) from late completion of the project.
- t_i :Time of realization of node *i* (AoA representation), where node 1 is the "start node" of the project and node *n* its "end node".
- T_s :Target completion time of the project.

Model Description

- $c_R(a, r_p)$: cost of resources for activity *a* resource r_p (including complementary resources).
- $c_R(a)$: cost of resources for activity *a* (includes all resources).
- e : earliness.
- d : tardiness (delay).
- c_E : cost of earliness.
- c_T : cost of tardiness.
- c_{ET}: cost of earliness and tardiness.
- TC: total cost.

Model Description

- Respect precedence among the activities:

$$t_j \geq t_i + y(a), \ \ \forall a \equiv (i, j) \in A$$

- Define total allocation of resource r_p (including complementary resource) in activity a,

$$x_a\left(r_p, \{s_q\}_{q=1}^{\sigma}\right) = x_a(r_p) + \sum_{q=1}^{\sigma} v(r_p, s_q) * x_a^{r_p}(s_q)$$

Model Description

- Define the duration of each activity when using each *P*-resource:

$$y_a(r_p, \{s_q\}_{q=1}^{\sigma}) = \frac{w_a(r_p)}{x_a\left(r_p, \{s_q\}_{q=1}^{\sigma}\right)}$$

- Define the activity's duration as the maximum of individual resource durations:

$$y(a) = \underbrace{max}_{all\ r_p}\left\{y_a(r_p, \{s_q\}_{q=1}^{\sigma})\right\}$$

Model Description

- Respect the *P*-resource availability at each *udc* traversed by the project in its execution,

$$\sum_{a \in C^k} x_a(r_p) \leq \mathrm{Q_P(p)}, \qquad \forall p \in P$$

Respect also the *S*-resource considering again the current *udc*,

$$\sum_{a \in C^k} x_a^{r_p}(s_q) \leq \mathrm{Q_S(q)} \quad \forall q \in S$$

Model Description

- Define earliness and tardiness by:

$$e \geq T_s - t_n$$

$$d \geq t_n - T_s$$

$$e, d \geq 0$$

Model Description

- The cost of resource utilization is:

$$c_R(a, r_p) = \left(\gamma_p * x_a(r_p) + \gamma_q * \sum_{q=1}^{\sigma} x_a^{r_p}(s_q) \right) * w(a, r_p)$$

$$c_R(a) = \sum_{\text{all } r_p} c_R(a, r_p)$$

Model Description

- The earliness-tardiness costs are linear in their respective marginal values.

$$c_{ET} = c_E + c_T = \gamma_E \cdot e + \gamma_L \cdot d$$

- The desired objective function may be written as:

$$\min TC = \sum_{a \in A} c_R(a) + c_{ET}$$

Description of the Procedure Adopted

- Description
  - The process begins by analyzing the network and the resource requirements and constructing the state space.
  - During the course of the project execution, each activity can be in one and only one of the following three states.

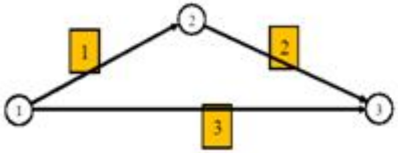
    (i) *Active*: an activity is active at time *t* if it is being executed at time *t*.

    (ii) *Dormant*: an activity is dormant at time *t* if it has finished but there is at least one unfinished activity that ends at the same node as *a*.

    (iii) *Idle*: an activity *a* is called idle at time *t* if it is neither active nor dormant at time *t*: the activity is either completed or is yet to be started.

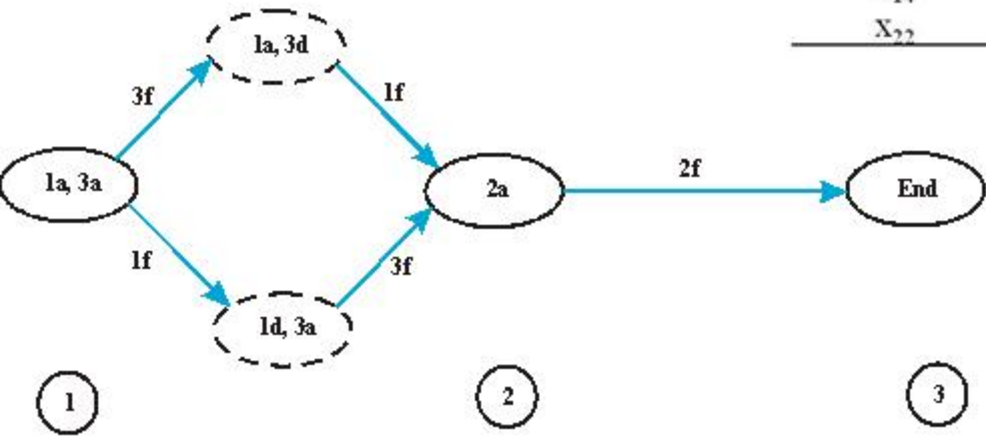
Description of the Procedure Adopted

- Application



Decision Variable	Possible Values	Decision Variable	Possible Values
x_{11}	1	x_{222}	0..2
x_{111}	0..1	x_{24}	1..2
x_{112}	0..2	x_{31}	1
x_{13}	1..3	x_{311}	0..1
x_{131}	0..1	x_{312}	0..2
x_{14}	1..2	x_{32}	1
x_{22}	1	x_{322}	0..2

x_{aps}
a: activity.
p: primary resource.
s: supportive resource.



State Space diagram for the activity network 1

Description of the Procedure Adopted

- We use a tridimensional structure to save the information as follows:

$$X[a, r, k]$$

  – Where:

    - *X:* Structure that will contain the information for each combination of resources considered;
    - *a:* Represent the activity;
    - *r:* Represent the (primary) resource;
    - *k:* Represent the kind of information stored;

Description of the Procedure Adopted

- We have six possibilities for the index *k*:

$$X[a,r,k]$$

  - If $k=1 \Rightarrow$ quantities of the primary resource of a combination.
  - If $k=2 \Rightarrow$ quantities of the *S*-resource 1.
  - If $k=3 \Rightarrow$ quantities of the *S*-resource 2.
  - If $k=4 \Rightarrow$ total quantity of resources (*P*- and *S*- resources) for each pair activity – *P*-resource.
  - If $k=5 \Rightarrow$ corresponding resource cost.
  - If $k=6 \Rightarrow$ duration of each pair activity – *P*-resource.

Description of the Procedure Adopted

- We will exemplify the evaluation of the values of X for one combination of the resources.

$X(k = 1)$

↓Activity/*P*-Resource→	1	2	3	4
1	1	x	2	2
2	x	1	x	2
3	1	1	x	x

$X(k = 2)$

↓Activity/*P*-Resource→	1	2	3	4
1	0	x	1	x
2	x	x	x	x
3	0	x	x	x

$X(k = 3)$

↓Activity/*P*-Resource→	1	2	3	4
1	0	x	x	x
2	x	0	x	x
3	0	1	x	x

Description of the Procedure Adopted

- Total quantity of resources

$$X[a,r,4] = X[a,r,1] + X[a,r,2] * v[1,r] + X[a,r,3] * v[2,r]$$

$X\,(\,k = 4\,)$

↓Activity/P-Resource→	1	2	3	4
1	1	x	2.25	2
2	x	1	x	2
3	1	1.35	x	x

Description of the Procedure Adopted

- Resource cost

$$X[a,r,5] = \left(X[a,r,1] * \gamma_p + X[a,r,2] * \gamma_q + X[a,r,3] * \gamma_q\right) * w(a,r)$$

$X(k=5)$

↓Activity/P-Resource→	1	2	3	4
1	64	0	108	96
2	0	28	0	64
3	80	110	0	0

Description of the Procedure Adopted

- Duration of each pair activity – *P*-resource

$$X[a,r,6] = \frac{w(a,r)}{X[a,r,4]}$$

X (k = 6)

↓Activity/*P*-Resource→	1	2	3	4
1	16	x	5.3	6.0
2	x	7.0	x	4.0
3	20	16.3	x	x

Description of the Procedure Adopted

- Final activities durations

$$y(1) = \max(16.0, 5.3, 6.0) = 16.0$$
$$y(2) = \max(7.0, 4.0) = 7.0$$
$$y(3) = \max(20.0, 16.3) = 20.0$$

Description of the Procedure Adopted

- Initially the state will be equal to 1.
- To proceed we have to evaluate if the combination is valid, considering the total availability of the resources until will achieve the final state.
- Considering Activity Network 1, we have:

$$t_1 = 0$$
$$t_2 = \max(16{,}20) = 20$$
$$t_3 = t_2 + y(2) = 20 + 7 = 27.$$

- The due date of the project is $Ts = 24$, so we will be 3 days late. The cost of tardiness will be,

$$C_T = 3 * \gamma_L = 3 * 60 = 180 \text{ as } C_E\!=\!0,\ C_{ET}\!=\!180.$$

Description of the Procedure Adopted

- The resource cost in the sum of all resource costs for all pairs activity – *P*-resource:

$$C_R = 64 + 108 + 96 + 28 + 64 + 80 + 110 = 550.$$

- The total cost *TC* is:

$$TC = C_{ET} + C_R = 180 + 550 = 730.$$

Results – Network 1

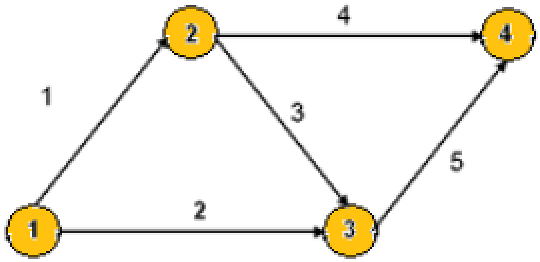
- After repeating this procedure for all possible combinations, we came up with a solution of this network, based on an initial implementation in C:

x11	x111	x112	x13	x131	x14	x22	x222	x24	x31	x311	x312	x32	x322
1	0	0	1	0	1	1	2	2	1	1	0	1	1

Tn	C_E	C_T	C_{ET}	C_R	TC
20.41	-143.44	0	-143.44	476	332.56

Results of Application – Network 2

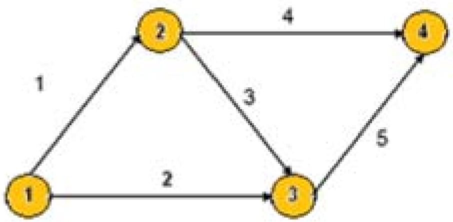
- Consider now the activity network 2



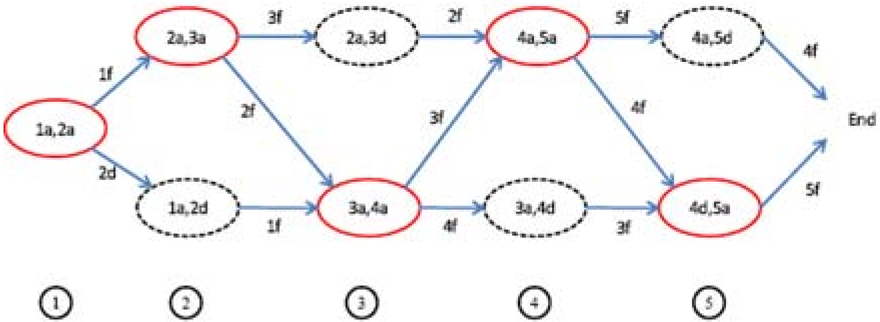
P-resource →	*1*	*2*	*3*	*4*
↓ Activity/Availability→	2	1	3	3
A1	16	0	12	12
A2	0	7	10	8
A3	20	0	22	0
A4	0	7	0	8
A5	20	0	16	0

Results of Application – Network 2

- Application



Decision Variable	Possible Values	Decision Variable	Possible Values
x_{11}	1	x_{312}	0..2
x_{111}	0..1	x_{33}	0..2
x_{112}	0..2	x_{331}	1..2
x_{13}	1..3	x_{42}	1
x_{131}	0..1	x_{422}	0..2
x_{14}	1..2	x_{44}	1..2
x_{22}	1	x_{51}	1..2
x_{222}	0..2	x_{511}	0..1
x_{24}	1..2	x_{512}	0..2
x_{31}	1	x_{53}	1..3
x_{311}	0..1	x_{531}	0..1



State Space diagram for the activity network 2

Results of Application – Network 2

- After repeating the same procedure for all possible combinations, we came up with a solution of network 2

x11	x111	x112	x13	x131	x14	x22	x222	x23	x231	x24	x31	x311	x312	x33	x331	x42	x422	x44	x51	x511	x512	x53	x531
2	1	2	2	0	2	1	0	1	0	1	2	0	0	2	1	1	0	1	2	1	1	2	0

Tn	CE	CT	CET	CR	TC
24.61	0.00	36.47	36.47	1214.00	1250.47

Conclusions

- The goal of this paper was to provide a formal model to some unresolved issues in the management of projects, especially as related to the utilization of supportive resources, and to its implementation.
- The relevance of the problem is the opportunity to shape a system that allows not only that we improve the allocation of often scarce resource(s), but also result in reduced uncertainties within the projects, combined with increased performance and lower project costs.

Conclusions

- The model was first presented in [3] but there remained its implementation and application to some project networks, to demonstrate its validity. In this paper we presented the procedure developed to solve the mathematical model and we applied it to two activity networks, obtaining the desired results, through an initial implementation in C.

Conclusions

- There still remains the implementation of the model in an easy-to-use computer code that renders it practically usable for networks of realistic size. This should be a general computer code, which will be capable calculating the solution for any activity network.
- Considering the feasibility of the model proposed, we believe it can provide to user a new option to plan and to determine the best combination of resources and the lowest project cost, pushing the planning phase and increase the estimation ability of the companies.

We thank Prof. Elmaghraby for his contribution in the definition of this problem.

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Thank you very much!!!