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Multiple Resources Allocation under Stochastic Conditions on Projects with Multimodal Activities

Rui Moutinho
Informatics Department
University of Minho
Braga, Portugal
rumout@gmail.com

Anabela Tereso
Production and Systems Department
University of Minho
Braga, Portugal
anabelat@dps.uminho.pt

Abstract
A "multi-modal" activity is one which can be performed in several modes, or levels, of resource(s) allocation. In this paper we assume that the activities are multi-modal, and each activity may require several resources for its execution. We also assume that the resources may be constrained on their availability over time, and that each activity has its own work contents as a continuous random variable for each of its required resources. We are concerned with establishing a suitable strategy that will focus on the universe of activity modes, selecting only those that will be relevant for the most probable overall project cost. In other words, we choose the most probable modes of activity execution among the theoretical continuum of modes; and propagate the consequences of that choice throughout the project. The proposed approach relies on the partitioning of the work content domains into classes. Using these classes, we address different probabilities for each feasible allocation.
Keywords: Project Management; Multiple Resource Allocation; Cost Minimization.

1. Introduction

In this article we address the allocation of multiple resources under multi-modal project activities with stochastic work content. Under these conditions, we specify a work content for each resource in an activity – W_r. Each work content is a continuous random variable exponentially distributed:

$$W_r \sim \text{Exp}(\lambda_r) \tag{1}$$

Thus, we have a continuous domain of execution modes for each activity.
One of the issues involved on such problems is which and how to select those resource allocations which yield, on all resources on the same activity, equal local durations, in expectation. Denote Y_r as the local duration yield by a resource r on an activity with R resources, we desire to allocate quantities such:

$$\mathcal{E}\left[Y_r\right] = \mathcal{E}\left[Y_s\right], \, r, s = \{1, \ldots, R\} \tag{2}$$

Where all the Y_r are evaluated with respect to the work content and the allocated quantity x_r:

$$Y_r = W_r / x_r \tag{3}$$

The activity duration Y is the maximum of those yield by each of its resources:

$$Y = \max\left\{Y_r | r = \{1, \ldots, R\}\right\} \tag{4}$$

The way the activity duration is evaluated stem the stated desire. The condition on Eq.(2) is made such no resource, by ceasing earlier, becomes idle, thus, incurring in wasteful allocation, in expectation. We already address this particular issue in other works. On [1] we show how to counterpart the waste with the insertion of a new factor – a maintenance cost – in the total project cost. Other approaches do not apply new factors, instead they devise suitable allocations from the permissible range of quantities which yield the desired equal durations [3] or devise the suitable allocations ranges from inspection on the possible equal durations [2]. The first mentioned was actually implemented and the other two shown not to be practical, yet they brought some insight to the allocation of multiple resources. In this article we will present yet another approach and study the sensitivity of the allocated quantities and number of resources into the desired equality. This approach relies on the partitioning of the innumerable number of activity modes into a finite set of specially selected modes that would represent the most probable activity durations.

1.1. Motivation
We have been using the exponential distribution to characterize the work content from a resource in an activity. Such distribution is continuous with `C.D.F.` (*cumulative distribution function*) given by:

$$P(W \leqslant w) = \begin{cases} 1 - e^{-w\lambda} & w > 0 \\ 0 & \text{otherwise} \end{cases}, \lambda > 0 \tag{5}$$

The case whereas the work content is zero is not under our analysis due to two reasons. First, the probability of the work content being zero is zero. Also, a zero work content would represent a dummy resource use in an activity. Therefore, we shall address the `C.D.F.` of each W_r, denoted as F_{W_r}, as:

$$F_{W_r}(w) = 1 - e^{-w\lambda_r} \tag{6}$$

1.2. Distribution Functions
Consider only the context of an activity with R resources. Each resource yields a local duration. The `C.D.F.` of each of the local durations secured by resource r, denoted as F_{Y_r}, is easily determined for an arbitrarily fixed allocation x_r by means of Eq.(6):

$$F_{Y_r} = P(Y_r \leqslant y) = P(W_r/x_r \leqslant y) = P(W_r \leqslant yx_r) = F_{W_r}(yx_r) = 1 - e^{-\lambda_r x_r y} \tag{7}$$

Since the resources contributions are taken as independent, one find the `C.D.F.` of the activity duration, denoted as F_Y, as:

$$F_Y(y) = \prod_{r=1}^{R} F_{Y_r}(y) \tag{8}$$

1.3. Finite Activity Modes
In the remainder of this article we will base our studies on partitioning of the $\mathbb{R}^+$ domain of the random variables into a finite number of equal amplitude classes: each class corresponds to a probability of occurrence. The reader may already notice that such finite partition is not straightforward since the original distribution is infinite. In order to achieve that we shall truncate the original distribution such that we retain nearly all the “significance”, say, 99% of the distribution.
We will express the class amplitudes by δ and the discarded percentage as ϵ. Notice that $\delta > 0$ and $\epsilon < 1$ (take 1 as 100%). On Fig. 1 we represent one partition of an exponential distribution (given its probability density function).

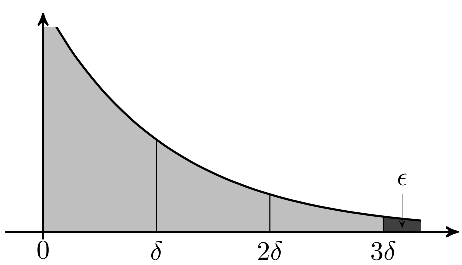

Figure 1: Partition of the exponential distribution. The area below the function line and above the x-axis is equal to 1. The domain partition, also divides that area into smaller ones. The area discarded is marked with darker shade.

The last (nth) class is in the form $](n-1)\delta, n\delta]$. The evaluation of the n value stems from the meaning of ϵ, namely, we want n such $n\delta$ is the first occurrence in the discarded domain. This can be done with simple algebra over a (exponential) `C.D.F.`, $F(.)$. First, the discarded domain beginning – x – is determined by:

$$F(x) = 1 - \epsilon \Leftrightarrow x = -\left(\ln \epsilon\right)/\lambda \tag{9}$$

Then, we find:

$$n\delta = -\left(\ln \epsilon\right)/\lambda \Leftrightarrow n = -\left(\ln \epsilon\right)/(\lambda\delta) \tag{10}$$

From which we conclude,

$$n = \left\lceil -\frac{\ln \epsilon}{\lambda\delta} \right\rceil \tag{11}$$

When we apply this partitioning process to the local durations of each activity resources, for an arbitrary fixed allocation vector, we find a finite number of execution modes for each resource; therefore, for the activity.

In the next section we will introduce two strategies on determining the most probable execution modes among those who would secure equal local durations.

2. Studying the Local Duration Equality

For an activity, we want to identify the execution modes combinations who secure equal local durations without any waste. Also, for each mode combination we are interested on the associated probability of occurrence.

In the remainder of the section we will present two approaches to the problem, focused on an arbitrary activity with an arbitrary number of resources R. Each resource r as an arbitrary fixated allocated quantity x_r.

2.1. Bottom-Up Approach

Fixating an allocation vector $(x_1, \ldots, x_R)$, all the F_{Y_r} can be evaluated, then partitioned according to specified δ and ϵ parameters. Denote by Δ_r the set of all the modes determined for resource r:

$$\Delta_r = \left\{](i-1)\delta, i\delta] \,|i \in \{1, \ldots, n_r\} \right\} \tag{12}$$

where n_r is evaluated according to Eq.(11) in reference to F_{Y_r}. Namely,

$$n_r = \left\lceil -\frac{\ln \epsilon}{\lambda_r x_r \delta} \right\rceil \tag{13}$$

Each mode, in the form $](i-1)\delta, i\delta]$, from resource r, has the probability – $p_{r,i}$ –, determined by:

$$p_{r,i} = P\left(Y_r \in](i-1)\delta, i\delta]\right) = F_{Y_r}(i\delta) - F_{Y_r}((i-1)\delta) \tag{14}$$

In order to avoid any "waste", we must select resource modes that yield the same durations, i.e. the same intervals. Due to the partition process, it is trivial to realize that, assuming the usual ordering on the intervals, the first k modes are identical on all the resources, where

$$k = \min\left\{\#\Delta_1, \ldots, \#\Delta_R\right\} \tag{15}$$

We are now able to construct a table with the activity durations, obtained without "waste", and its probabilities. Denoting by Y^i the activity duration when using the ith mode of each resource and by p_i the associated probability of Y^i, we evaluate:

$$\forall i \in \{1, \ldots, k\}, \, Y^i =](i-1)\delta, i\delta] \wedge p_i = \prod_{r=1}^{R} p_{r,i} \tag{16}$$

As an example, take one activity with two resources such:

$$\begin{array}{ll} \lambda_1 = 1 & x_1 = 2 \\ \lambda_2 = 3 & x_2 = 1 \end{array} \Rightarrow \begin{array}{l} F_{Y_1}(x) = 1 - e^{-2x} \\ F_{Y_2}(x) = 1 - e^{-3x} \end{array} \tag{17}$$

Intuitively, one may expect the use of small δ and ϵ parameters. However, in order to shorten the illustrative example, take $\delta = 0.3$ and $\epsilon = 0.1$. Now, we can proceed with the partitions by first evaluating the n_r:

$$n_1 = \left\lceil -\frac{\ln 0.1}{0.6} \right\rceil = 4 \quad n_2 = \left\lceil -\frac{\ln 0.1}{0.9} \right\rceil = 3 \tag{18}$$

Which leads to the sets:

$$\Delta_1 = \Big\{]0, 0.3] \, ,]0.3, 0.6] \, ,]0.6, 0.9] \, ,]0.9, 1.2] \Big\} \tag{19}$$

$$\Delta_2 = \Big\{]0, 0.3] \, ,]0.3, 0.6] \, ,]0.6, 0.9] \Big\} \tag{20}$$

Finally, we evaluate the Y^i and their probabilities:

$$\begin{array}{ll} Y^1 =]0, 0.3] & p_1 = (F_{Y_1}(0.3) - F_{Y_1}(0)) \times (F_{Y_2}(0.3) - F_{Y_2}(0)) = 0.2677 \\ Y^2 =]0.3, 0.6] & p_2 = (F_{Y_1}(0.6) - F_{Y_1}(0.3)) \times (F_{Y_2}(0.6) - F_{Y_2}(0.3)) = 0.0597 \\ Y^3 =]0.6, 0.9] & p_3 = (F_{Y_1}(0.9) - F_{Y_1}(0.6)) \times (F_{Y_2}(0.9) - F_{Y_2}(0.6)) = 0.0133 \end{array} \tag{21}$$

2.2. Top-Down Approach

Instead of partitioning the local durations, we may apply the same process directly over the activity duration by means of its `C.D.F.`, on Eq.(8) on page 2. Denote by Δ_Y the set of all the classes obtained from partitioning the activity duration, accordingly to a δ and a ϵ parameters.

$$\Delta_Y = \Big\{](i-1)\delta, i\delta] \, | i \in \{1, \ldots, n\} \Big\} \tag{22}$$

where n is obtained according to Eq.(11) on the previous page but, in this case, not easy to simplify. The n is the first positive integer value securing:

$$F_Y(n\delta) \geqslant 1 - \epsilon \tag{23}$$

Reusing the notation from the previous approach we have, directly, the $Y^i \in \Delta_Y$. The probability associated to each Y^i – the p_i – can not be evaluated from the F_Y `C.D.F.` as such value refers to the probability of the maximum of the Y_r. Notice that each Y^i interval may be the result (trough maximum) of several possible Y_r intervals; not only those with no "waste". Therefore, we shall use each F_{Y_r} `C.D.F.` in order to evaluate the desired values:

$$\forall Y^i =](i-1)\delta, i\delta] \in \Delta_Y, \, p_i = \prod_{r=1}^{R} (F_{Y_r}(i\delta) - F_{Y_r}((i-1)\delta)) \tag{24}$$

Recalling the same example given for the bottom-up approach, the `C.D.F.` for the activity duration is

$$F_Y(x) = F_{Y_1}(x) \times F_{Y_2}(x) \tag{25}$$

And, we construct the Δ_Y set by starting with the 1st element – $]0, \delta]$ – and iterating until the Eq.(23) is satisfied. For our example, it results in:

$$\Delta_Y = \Big\{]0, 0.3],]0.3, 0.6],]0.6, 0.9],]0.9, 1.2],]1.2, 1.5] \Big\} \tag{26}$$

We conclude by evaluating the related probabilities:

$$\begin{array}{ll} Y^1 =]0, 0.3] & p_1 = (F_{Y_1}(0.3) - F_{Y_1}(0)) \times (F_{Y_2}(0.3) - F_{Y_2}(0)) = 0.2677 \\ Y^2 =]0.3, 0.6] & p_2 = (F_{Y_1}(0.6) - F_{Y_1}(0.3)) \times (F_{Y_2}(0.6) - F_{Y_2}(0.3)) = 0.0597 \\ Y^3 =]0.6, 0.9] & p_3 = (F_{Y_1}(0.9) - F_{Y_1}(0.6)) \times (F_{Y_2}(0.9) - F_{Y_2}(0.6)) = 0.0133 \\ Y^4 =]0.9, 1.2] & p_4 = (F_{Y_1}(1.2) - F_{Y_1}(0.9)) \times (F_{Y_2}(1.2) - F_{Y_2}(0.9)) = 0.00297 \\ Y^5 =]1.2, 1.5] & p_5 = (F_{Y_1}(1.5) - F_{Y_1}(1.2)) \times (F_{Y_2}(1.5) - F_{Y_2}(1.2)) = 0.00066 \end{array} \tag{27}$$

3. Results

Before presenting the results, we shall point out some aspects concerning the testing of the two approaches. First, although each approach is applicable to any continuous probability distribution, we will stay in focus on the particular case of the exponential distribution. Under this context, it is easy to realize that it is not necessary to study combinations of values λ and allocated quantity x for the same resource. One can simply set λ equal to 1 and vary the allocated quantity, because, for the approaches presented, it is only relevant the result of λx. On the other hand, say, having $\lambda = .1$ and $x = 3$ does not mean, in practice, the same as $\lambda = 0.15$ and $x = 2$; despite being $\lambda x = 0.3$ in both cases. Thus, we will deliberately set $\lambda = 1$ only for convenience in the testing and results presentation.
Last, we will "measure" the obtained results obtained in each test by:

- The total sum of the p_i values, denoted as P. This will be taken as the probability of occur no wasteful allocations;
- The sample mean of the Y^i with respect to their probabilities and taking the total p_i sum as the unit. We denote this mean as $\dot{Y}$ and evaluate it as:

$$\dot{Y} = \frac{\sum_i \left(\dot{Y}^i p_i\right)}{\sum_i p_i} \tag{28}$$

  where $\dot{Y}^i$ is the mark of the class Y^i:

$$Y^i =]a, b] \Rightarrow \dot{Y}^i = (a + b)/2 \tag{29}$$

- The mean of the distribution of the activity duration Y, denoted as $\overline{Y}$:

$$\overline{Y} = \int_0^{+\infty} x(F_Y(x))' \, \mathrm{d}x \tag{30}$$

Along with the above, we will include a measurement of the difference between the $\dot{Y}$ and $\overline{Y}$ values, evaluated in percentage in respect to $\overline{Y}$. On this regard, we take, arbitrarily, the $\dot{Y}$ value obtained by the Bottom-Up approach because it is predictable that the two approaches will result in very similar results. However, we will be sensitive to this choice if the results show otherwise. Denoting that measure as d:

$$d = \frac{\overline{Y} - \dot{Y}}{\overline{Y}} \times 100 \tag{31}$$

In the following, we present the results from the tests we have made. For all tests, $\delta = 0.05$ and $\epsilon = 0.00000001$ where used on both approaches.

3.1. Resource Multiplicity

With this set of tests we want to observe the implications from the resource multiplicity. Therefore, we use always equal allocated quantities. On all the tables, we distinguish the results from the two approaches by using their initials as index on the respective values.

Table 1: Context of one resource with allocated quantities ranging from 0.5 to 2 in steps of 0.25.

X	$\dot{Y}_{BU}$	P_{BU}	$\dot{Y}_{TD}$	P_{TD}	$\overline{Y}$	d
(0.5)	2.0001	1.0	2.0001	1.0	2.0	0.0052%
(0.75)	1.3335	1.0	1.3335	1.0	1.3333	0.0117%
(1.0)	1.0002	1.0	1.0002	1.0	1.0	0.0208%
(1.25)	0.8003	1.0	0.8003	1.0	0.8	0.0325%
(1.5)	0.667	1.0	0.667	1.0	0.6667	0.0469%
(1.75)	0.5718	1.0	0.5718	1.0	0.5714	0.0638%
(2.0)	0.5004	1.0	0.5004	1.0	0.5	0.0833%

Table 2: Two resources with allocated quantities all ranging from 0.5 to 2 with steps of 0.25.

X	$\dot{Y}_{\mathbf{BU}}$	$P_{\mathbf{BU}}$	$\dot{Y}_{\mathbf{TD}}$	$P_{\mathbf{TD}}$	$\overline{Y}$	d
$(0.5, 0.5)$	1.0002	0.0125	1.0002	0.0125	3.0	66.6597%
$(0.75, 0.75)$	0.667	0.0187	0.667	0.0187	2.0	66.651%
$(1.0, 1.0)$	0.5004	0.025	0.5004	0.025	1.5	66.6389%
$(1.25, 1.25)$	0.4005	0.0312	0.4005	0.0312	1.2	66.6233%
$(1.5, 1.5)$	0.334	0.0375	0.334	0.0375	1.0	66.6042%
$(1.75, 1.75)$	0.2864	0.0437	0.2864	0.0437	0.8571	66.5816%
$(2.0, 2.0)$	0.2508	0.05	0.2508	0.05	0.75	66.5556%

Table 3: Three resources with allocated quantities all ranging from 0.5 to 2 with steps of 0.25.

X	$\dot{Y}_{\mathbf{BU}}$	$P_{\mathbf{BU}}$	$\dot{Y}_{\mathbf{TD}}$	$P_{\mathbf{TD}}$	$\overline{Y}$	d
$(0.5, 0.5, 0.5)$	0.667	0.0002	0.667	0.0002	3.6667	81.8097%
$(0.75, 0.75, 0.75)$	0.4449	0.0005	0.4449	0.0005	2.4444	81.799%
$(1.0, 1.0, 1.0)$	0.334	0.0008	0.334	0.0008	1.8333	81.7841%
$(1.25, 1.25, 1.25)$	0.2674	0.0013	0.2674	0.0013	1.4667	81.7649%
$(1.5, 1.5, 1.5)$	0.2232	0.0019	0.2232	0.0019	1.2222	81.7415%
$(1.75, 1.75, 1.75)$	0.1916	0.0025	0.1916	0.0025	1.0476	81.7139%
$(2.0, 2.0, 2.0)$	0.1679	0.0033	0.1679	0.0033	0.9167	81.682%

Table 4: Four resources with allocated quantities all ranging from 0.5 to 2 with steps of 0.25.

X	$\dot{Y}_{\mathbf{BU}}$	$P_{\mathbf{BU}}$	$\dot{Y}_{\mathbf{TD}}$	$P_{\mathbf{TD}}$	$\overline{Y}$	d
$(0.5, 0.5, 0.5, 0.5)$	0.5004	0	0.5004	0	4.1667	87.99%
$(0.75, 0.75, 0.75, 0.75)$	0.334	0	0.334	0	2.7778	87.9775%
$(1.0, 1.0, 1.0, 1.0)$	0.2508	0	0.2508	0	2.0833	87.96%
$(1.25, 1.25, 1.25, 1.25)$	0.201	0.0001	0.201	0.0001	1.6667	87.9376%
$(1.5, 1.5, 1.5, 1.5)$	0.1679	0.0001	0.1679	0.0001	1.3889	87.9101%
$(1.75, 1.75, 1.75, 1.75)$	0.1443	0.0002	0.1443	0.0002	1.1905	87.8777%
$(2.0, 2.0, 2.0, 2.0)$	0.1267	0.0002	0.1267	0.0002	1.0417	87.8404%

3.2. Allocation Variability

We tested the approaches, in a context of two resources, when the possible allocated quantities are combined in all the possible ways, in respect to a constant step. Each cell in the tables represents the results of each approach as a pair $(\dot{Y}, P)$ indexed by the initials of the corresponding approach, followed by the $\overline{Y}$ and the d.

Table 5: Two resources with allocated quantities varying in steps of 0.25 in all combinations. The columns vary the resource 2 allocation and the rows vary those of resource 1.

X	**0.5**	**0.75**	**1.0**	**1.25**
0.5	$(1.0002, 0.0125)_{\text{BU}}$ $(1.0002, 0.0125)_{\text{TD}}$ 3.0; 66.6597%	$(0.8003, 0.015)_{\text{BU}}$ $(0.8003, 0.015)_{\text{TD}}$ 2.5333; 68.4108%	$(0.667, 0.0167)_{\text{BU}}$ $(0.667, 0.0167)_{\text{TD}}$ 2.3333; 71.4152%	$(0.5718, 0.0179)_{\text{BU}}$ $(0.5718, 0.0179)_{\text{TD}}$ 2.2286; 74.3426%
0.75	$(0.8003, 0.015)_{\text{BU}}$ $(0.8003, 0.015)_{\text{TD}}$ 2.5333; 68.4108%	$(0.667, 0.0187)_{\text{BU}}$ $(0.667, 0.0187)_{\text{TD}}$ 2.0; 66.651%	$(0.5718, 0.0214)_{\text{BU}}$ $(0.5718, 0.0214)_{\text{TD}}$ 1.7619; 67.5469%	$(0.5004, 0.0234)_{\text{BU}}$ $(0.5004, 0.0234)_{\text{TD}}$ 1.6333; 69.3622%
1.0	$(0.667, 0.0167)_{\text{BU}}$ $(0.667, 0.0167)_{\text{TD}}$ 2.3333; 71.4152%	$(0.5718, 0.0214)_{\text{BU}}$ $(0.5718, 0.0214)_{\text{TD}}$ 1.7619; 67.5469%	$(0.5004, 0.025)_{\text{BU}}$ $(0.5004, 0.025)_{\text{TD}}$ 1.5; 66.6389%	$(0.4449, 0.0278)_{\text{BU}}$ $(0.4449, 0.0278)_{\text{TD}}$ 1.3556; 67.1785%
1.25	$(0.5718, 0.0179)_{\text{BU}}$ $(0.5718, 0.0179)_{\text{TD}}$ 2.2286; 74.3426%	$(0.5004, 0.0234)_{\text{BU}}$ $(0.5004, 0.0234)_{\text{TD}}$ 1.6333; 69.3622%	$(0.4449, 0.0278)_{\text{BU}}$ $(0.4449, 0.0278)_{\text{TD}}$ 1.3556; 67.1785%	$(0.4005, 0.0312)_{\text{BU}}$ $(0.4005, 0.0312)_{\text{TD}}$ 1.2; 66.6233%
1.5	$(0.5004, 0.0187)_{\text{BU}}$ $(0.5004, 0.0187)_{\text{TD}}$ 2.1667; 76.9038%	$(0.4449, 0.025)_{\text{BU}}$ $(0.4449, 0.025)_{\text{TD}}$ 1.5556; 71.3984%	$(0.4005, 0.03)_{\text{BU}}$ $(0.4005, 0.03)_{\text{TD}}$ 1.2667; 68.3799%	$(0.3642, 0.0341)_{\text{BU}}$ $(0.3642, 0.0341)_{\text{TD}}$ 1.103; 66.981%
1.75	$(0.4449, 0.0194)_{\text{BU}}$ $(0.4449, 0.0194)_{\text{TD}}$ 2.127; 79.0824%	$(0.4005, 0.0262)_{\text{BU}}$ $(0.4005, 0.0262)_{\text{TD}}$ 1.5048; 73.3831%	$(0.3642, 0.0318)_{\text{BU}}$ $(0.3642, 0.0318)_{\text{TD}}$ 1.2078; 69.8451%	$(0.334, 0.0364)_{\text{BU}}$ $(0.334, 0.0364)_{\text{TD}}$ 1.0381; 67.8297%
2.0	$(0.4005, 0.02)_{\text{BU}}$ $(0.4005, 0.02)_{\text{TD}}$ 2.1; 80.9276%	$(0.3642, 0.0273)_{\text{BU}}$ $(0.3642, 0.0273)_{\text{TD}}$ 1.4697; 75.2188%	$(0.334, 0.0333)_{\text{BU}}$ $(0.334, 0.0333)_{\text{TD}}$ 1.1667; 71.375%	$(0.3084, 0.0384)_{\text{BU}}$ $(0.3084, 0.0384)_{\text{TD}}$ 0.9923; 68.924%

Table 6: Continuation of the Tab. 5.

X	**1.5**	**1.75**	**2.0**
0.5	$(0.5004, 0.0187)_{\text{BU}}$ $(0.5004, 0.0187)_{\text{TD}}$ 2.1667; 76.9038%	$(0.4449, 0.0194)_{\text{BU}}$ $(0.4449, 0.0194)_{\text{TD}}$ 2.127; 79.0824%	$(0.4005, 0.02)_{\text{BU}}$ $(0.4005, 0.02)_{\text{TD}}$ 2.1; 80.9276%
0.75	$(0.4449, 0.025)_{\text{BU}}$ $(0.4449, 0.025)_{\text{TD}}$ 1.5556; 71.3984%	$(0.4005, 0.0262)_{\text{BU}}$ $(0.4005, 0.0262)_{\text{TD}}$ 1.5048; 73.3831%	$(0.3642, 0.0273)_{\text{BU}}$ $(0.3642, 0.0273)_{\text{TD}}$ 1.4697; 75.2188%
1.0	$(0.4005, 0.03)_{\text{BU}}$ $(0.4005, 0.03)_{\text{TD}}$ 1.2667; 68.3799%	$(0.3642, 0.0318)_{\text{BU}}$ $(0.3642, 0.0318)_{\text{TD}}$ 1.2078; 69.8451%	$(0.334, 0.0333)_{\text{BU}}$ $(0.334, 0.0333)_{\text{TD}}$ 1.1667; 71.375%
1.25	$(0.3642, 0.0341)_{\text{BU}}$ $(0.3642, 0.0341)_{\text{TD}}$ 1.103; 66.981%	$(0.334, 0.0364)_{\text{BU}}$ $(0.334, 0.0364)_{\text{TD}}$ 1.0381; 67.8297%	$(0.3084, 0.0384)_{\text{BU}}$ $(0.3084, 0.0384)_{\text{TD}}$ 0.9923; 68.924%
1.5	$(0.334, 0.0375)_{\text{BU}}$ $(0.334, 0.0375)_{\text{TD}}$ 1.0; 66.6042%	$(0.3084, 0.0404)_{\text{BU}}$ $(0.3084, 0.0404)_{\text{TD}}$ 0.9304; 66.8564%	$(0.2864, 0.0428)_{\text{BU}}$ $(0.2864, 0.0428)_{\text{TD}}$ 0.881; 67.4848%
1.75	$(0.3084, 0.0404)_{\text{BU}}$ $(0.3084, 0.0404)_{\text{TD}}$ 0.9304; 66.8564%	$(0.2864, 0.0437)_{\text{BU}}$ $(0.2864, 0.0437)_{\text{TD}}$ 0.8571; 66.5816%	$(0.2674, 0.0466)_{\text{BU}}$ $(0.2674, 0.0466)_{\text{TD}}$ 0.8048; 66.7669%
2.0	$(0.2864, 0.0428)_{\text{BU}}$ $(0.2864, 0.0428)_{\text{TD}}$ 0.881; 67.4848%	$(0.2674, 0.0466)_{\text{BU}}$ $(0.2674, 0.0466)_{\text{TD}}$ 0.8048; 66.7669%	$(0.2508, 0.05)_{\text{BU}}$ $(0.2508, 0.05)_{\text{TD}}$ 0.75; 66.5556%

4. Discussion

As we foresaw, the differences between the two approaches appear as negligible. This can be explained as the two approaches behave exactly the same for the common (initial) classes. The Top-Down approach can result in more classes than those of the Bottom-Up approach, with the same δ and ϵ. However, those extra classes, come associated with quite smaller probability. Thus little influence bring to the evaluation of the $\dot{Y}$.

Another observation is that not only the relative distance between the $\dot{Y}$ and the $\overline{Y}$ increases with the number of resources involved, but also the P decreases. We may conclude that the resource multiplicity renders the desire for non-wasteful allocations as a difficult task. Notice that this does not apply for the single resource case. This case is different from the rest because the C.D.F. has a completely different behavior (simple exponential distribution) from the C.D.F. with multiple resources (independent contribution of several random variables). We can observe that an increase on the quantity allocated to the resources (either one or more) seems to increase the P. However, this can be explained by the use of δ parameter. Realize that when λx increases, the C.D.F. "contracts", i.e., in the same domain, the larger the λx, the more representative the delimited area becomes, relatively to the total area. This means that one must not conclude that increase on the resource allocations brings more probability of the local durations being equal. We simply realize that the δ should be made smaller, in order to obtain a representative partition.

5. Conclusion

In this paper we propose two approaches on partitioning the total multi-modal domain of activities with multiple resources, under stochastic conditions. The goal is to observe on the implications of the resource multiplicity and allocated quantities in the desire of non-wasteful allocations – those which result in all equal durations yield by all the resources in an activity.

We observe that the two approaches produce very similar results. We also conclude that the non-wasteful allocations are sensitive to the amount of resources. Also, it seems much more important the correct choice of the partition parameters than the allocated quantity of the resources in the final probability of non-waste.

The desire for non-wasteful allocations comes from the need of avoid idleness on resources that were allocated to an activity. With the conclusions we made, it seems that one should consider replace that desire by the assumption or condition that a resource that is no longer needed by an activity, could be reassigned to another; thus, not becoming idle. Of course, this brings an all new set of consequences which are out of this article discussion.

Our main goal with the study presented in this paper, was to devise a suitable method for the characterization of the distribution of all the activities' durations, with no-waste. Then, such distribution would be used to evaluate the remain of the involved times; typically, the realization times of the nodes on the AoA (*Activity-on-Arc*) network mapping the precedence relations of the project activities. Ultimately, we would get a final distribution for the project duration along with the total cost. Despite our early intentions and facing these conclusions, we decided to rely on a different approach. Instead of devising a mathematical model using the work content distributions we follow the same guidelines used with WBras (*Waste Balance Resource Allocation Strategy*) [1] when dealing with the stochastic work content and non-wasteful allocations. Furthermore, we introduce the possibility for the activities to be start-delayed in order to cope with the resources availabilities.

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