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Duration Oriented Resource Allocation Strategy on Multiple Resources Projects under Stochastic Conditions ⋆

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Abstract

Previous developments from the first author and other researchers were made on devising models for the total cost optimization of projects described by activity networks under stochastic conditions. Those models only covered the single resource case.

The present paper will discuss the case of multiple resources. More precisely, we introduce a strategy of allocation of those resources in order to minimize the waste arising from their latent idleness on their consumption within the same activity. With this strategy we will start from the possible durations yielded by each resource and devise the allocation vector leading to equal durations.

Key words: Project Management and Scheduling, Stochastic Activity Networks, Resource Allocation, Multiple Resources

List of Acronyms

RCPSP: *Resource Constraint Project Scheduling Problem*
AoA: *Activity-on-Arc*
r.v.: random variable
DP: *Dynamic Programming*
EMA: *Electromagnetic Algorithm*
EVA: *Evolutionary Algorithm*
SRpco: *Single Resource Project Cost Optimization*
MRpco: *Multiple Resources Project Cost Optimization*
DOras: *Duration Oriented Resource Allocation Strategy*
WBras: *Waste Balance Resource Allocation Strategy*

1 Introduction

This paper follows the research made by several contributions starting with the research by the first author (see [1]). These works address a version of the RCPSP (*Resource Constraint Project Scheduling Problem*) in which we wish to determine the optimal allocation of resources to the project's multimodal activities that minimizes the total project cost under stochastic conditions.

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We adopt the AoA (*Activity-on-Arc*) network representation of the project. By 'multimodal activity' we mean an activity which duration is a function of the resource allocation. We assume that stochasticity resides in the work content of the activity, denoted by W, which is a *r.v.* (*random variable*). The first attempt at the resolution of this problem proposed a DP (*Dynamic Programming*) model [2] which was implemented in MATLAB. Subsequent research improved the computational aspects of the model by migrating it to JAVA [6]. Another MATLAB implementation using the EMA (*Electromagnetic Algorithm*) [3] was also migrated to JAVA [7]. Then, the use of the EVA (*Evolutionary Algorithm*) was applied to the same problem [5].

All these implementations, however, treated only the SRPCO (*Single Resource Project Cost Optimization*) case where the total project cost C is specified as

$$C = \mathcal{E}\left[\sum_{a\in \mathrm{A}} (c \times x_a \times W_a) + c_L \times \max\left(0, \Upsilon_n - T\right)\right] \tag{1}$$

where the following notation applies:

A: Set of project activities;
c: Quantity (of resource) cost per unit
c_L: Project delay cost per time unit
$x_a \in \mathbb{R}^+$: Allocated quantity of resource on activity a such $l_a \leqslant x_a \leqslant u_a$
$W_a \sim \mathrm{Exp}(\lambda_a)$: Work content of activity a
Υ_n: Evaluated realization time of last node
T: Schedule project realization time

In this paper we address the MRPCO (*Multiple Resources Project Cost Optimization*) problem in which each activity may require several resources for its execution. Each activity a will have its work content, W_r^a for resource r, $r \in \mathrm{R}_a$, and its allocation constraints according to its needs.

The immediate impact of resource multiplicity is to modify the total project cost C of Eq. (1) to

$$C = \mathcal{E}\left[\sum_{a\in \mathrm{A}} \sum_{r\in \mathrm{R}_a} (c_r \times x_r^a \times W_r^a) + c_L \times \max\left(0, \Upsilon_n - T\right)\right] \tag{2}$$

where the extended notation applies:

R_a: project resources subset needed by activity a
c_r: quantity cost per unit of resource r
$x_r^a \in \mathbb{R}^+$: allocated quantity of resource r on activity a such $l_r^a \leqslant x_r^a \leqslant u_r^a$
$W_r^a \sim \mathrm{Exp}(\lambda_r^a)$: Work content of resource r on activity a

To each resource allocation to an activity is associated an individual duration Y_r^a evaluated by

$$Y_r^a = \frac{W_r^a}{x_r^a} \tag{3}$$

from which we deduce that the actual activity duration, Y_a, is the maximum:

$$Y_a = \max_{r\in \mathrm{R}_a}\left(Y_r^a\right) \tag{4}$$

Clearly, Y_r^a is a *r.v.*, so is Y_a. Since it makes little sense to expend more of a resource (and incur higher cost) to have the activity duration under this resource shorter than its duration under any other resource, it is desired to have all durations equal. The probabilistic nature of the Y_r^a's forces us to pose this condition in terms of expectations; thus we require that

$$\mathcal{E}\left[Y_r^a\right] = \mathcal{E}\left[Y_s^a\right], \forall r, s \in \mathrm{R}_a \text{ and } \forall a \in \mathrm{A} \tag{5}$$

To determine the allocation vectors that satisfy such equality we propose the following strategy, which we label as DORAS (*Duration Oriented Resource Allocation Strategy*).

2 Duration Oriented Strategy

Instead of trying to equalize the individual expected durations through manipulation of the allocation vector X to satisfy Eq. (5) the DORAS determines which are the possible equal durations, then, it goes backwards and devises the allocation vectors yielding those durations.

For each activity, we first evaluate the intervals of feasible durations yielded by each resource alone. Then, the analysis of the intersection of such intervals leads to a final duration interval – the possible equal durations – and the resources contributing to such durations. This then determines the allocation vectors satisfying Eq. (5).

In order to enable more flexibility in the study of this new strategy, the DORAS works directly with sets of sample values instead of only the expected value. The explanation of the DORAS procedure is given by a sequence of running examples which provide both a demonstration of the steps of the procedure as well as an analysis and discussion of the procedure itself.

Example – Part 1 of 8

We start this running example for DORAS by supposing one activity $a \in \mathrm{A}$ with three resources indexed by integer numbers from 1 to 3 ($\#\mathrm{R}_a = 3$). Below, the initial parameters are listed.

$$x_i^a \in [0.5, 1.5]\,,\ i \in \{1,2,3\} \qquad \lambda_1^a = 0.1 \qquad \lambda_2^a = 0.2 \qquad \lambda_3^a = 0.01 \tag{6}$$

From those distribution parameters we evaluate the following 4-value samples of each W_r^a.

$$\begin{aligned} \mathcal{W}_1^a &= \{1.37, 4.77, 10, 23.86\} \\ \mathcal{W}_2^a &= \{0.68, 2.38, 5, 11.93\} \\ \mathcal{W}_3^a &= \{13.7, 47.68, 100, 238.63\} \end{aligned} \tag{7}$$

Once the work content of each resource is sampled, we evaluate the range of possible durations for each resource on each activity. These durations are determined when each sampled work content value of a resource is fixed and all the possible allocations are used (of the same resource). The sampled are sets of real intervals, each for every fixed value of $\mathcal{W}_r^a$,

$$\mathcal{Y}_r^a = \left[\frac{\omega_r^a}{u_r^a}, \frac{\omega_r^a}{l_r^a}\right], \ \omega_r^a \text{ is a realization of } W_r^a,\ \forall r \in \mathrm{R}_a,\ \forall a \in \mathrm{A} \tag{8}$$

For instance, the realization $\omega_1^a = 1.37$ given above would give rise to (recall that $0.5 \leqslant x_1^a \leqslant 1.5$),

$$\mathcal{Y}_1^a = \left[\frac{1.37}{1.5}, \frac{1.37}{0.5}\right] = [0.91, 2.74] \tag{9}$$

The sample of the W_r^a's given in Eq. (7) gives rise to the following sets of intervals:

$$\begin{aligned} \{\mathcal{Y}_1^a\} &= \{[0.91, 2.74]\,; [3.18, 9.54]\,; [6.67, 20]\,; [15.91, 47.72]\} \\ \{\mathcal{Y}_2^a\} &= \{[0.46, 1.37]\,; [1.59, 4.77]\,; [3.33, 10]\,; [7.95, 23.86]\} \\ \{\mathcal{Y}_3^a\} &= \{[9.13, 27.40]\,; [31.78, 95.35]\,; [66.67, 200]\,; [159.09, 477.26]\} \end{aligned} \tag{10}$$

For ease on notation, all sets have their values in the same order as those within each $\mathcal{W}_r^a$.

Having established the individual duration intervals corresponding to the sampled work contents, we next seek the values that are common to all three resources since this would lead to the *simultaneous* satisfaction of the equality of Eq. (5).

Let $\Psi_r = \{\psi_{r,k}\}$ denote the set of intervals secured for resource r, $\forall r \in \mathrm{R}_a$ in which $\psi_{r,k}$ is the individual interval. For instance,

$$\psi_{1,1} = [0.91, 2.74]\,;\ \psi_{1,2} = [3.18, 9.54]\,;\ \psi_{1,3} = [6.67, 20]\,;\ \psi_{1,4} = [15.91, 47.72] \tag{11}$$

and

$$\Psi_1 = \{\psi_{r,k}\}_{k=1}^{4} = \{\psi_{1,1}, \psi_{1,2}, \psi_{1,3}, \psi_{1,4}\} \tag{12}$$

The desired result is defined by

$$\mathcal{Y}_a = \bigcap_r \Psi_r = \bigcap_{r,k} \psi_{r,k} \tag{13}$$

Example – Part 2 of 8

Continuing the running example, we can easily find the sample values of $\mathcal{Y}_a$. This task becomes rather laborious to execute by hand since the total number of intersections to be considered is equal to $4^3 = 64$. Table 1 exhibits some of those 64 evaluations.

Table 1
DORAS – Some sample values of $\mathcal{Y}_a$ evaluated by simple intersection

$\omega_1 \in \mathcal{W}_1^a$	$\omega_2 \in \mathcal{W}_2^a$	$\omega_3 \in \mathcal{W}_3^a$	$\psi_1 \in \mathcal{Y}_1^a$	$\psi_2 \in \mathcal{Y}_2^a$	$\psi_3 \in \mathcal{Y}_3^a$	$\psi = \psi_1 \cap \psi_2 \cap \psi_3$
1.37	0.68	13.7	$[0.91, 2.74]$	$[0.45, 1.36]$	$[9.13, 27.4]$	$\emptyset$
4.77	0.68	13.7	$[3.18, 9.54]$	$[0.45, 1.36]$	$[9.13, 27.4]$	$\emptyset$
10.0	0.68	13.7	$[6.67, 20.0]$	$[0.45, 1.36]$	$[9.13, 27.4]$	$\emptyset$
23.86	0.68	13.7	$[15.91, 47.72]$	$[0.45, 1.36]$	$[9.13, 27.4]$	$\emptyset$
4.77	5.0	13.7	$[3.18, 9.54]$	$[3.33, 10.0]$	$[9.13, 27.4]$	$[9.13, 9.54]$
10.0	5.0	13.7	$[6.67, 20.0]$	$[3.33, 10.0]$	$[9.13, 27.4]$	$[9.13, 10.0]$
4.77	11.93	13.7	$[3.18, 9.54]$	$[7.95, 23.86]$	$[9.13, 27.4]$	$[9.13, 9.54]$

Despite the simplicity (regardless of its computational burden) of the determination of the intersection of two or more intervals, a new problem arises when the intersection is null. Such occurrence is the norm rather than the exception and cannot be ignored because of its implication on the costs incurred since smaller durations usually imply greater cost of the resources allocation. Coping with this situation requires modification of Eq. (13). Insight into the required modification is gained by reference to our running numerical example.

Example – Part 3 of 8

From the first row on Table 1, we can see one case where the intersection is null. A closer look into the same row shows that, for example, $\psi_{1,1} \cap \psi_{2,1} \neq \emptyset$. This allows a new interpretation: resource 1 and resource 2 can ensure equal individual durations while all three resources cannot. Thus, by testing each one of the configurations (resources involved) we can check whether or not a (sub)set of resources can yield non-null intersections instead of just consider the case with all resources. In other words, for each combination of sample values of the $\mathcal{Y}_r^a$'s we arrange them in all possible tuples (without repetition) and check their intersection.

Table 2 compiles one search for all the non-null intersections among all the different configurations for the case reported on the first row of Table 1.

From the seven evaluated intersections in Table 2, four are non-null. Also, by writing each configuration as a set, the set of all the configurations contains $2^3 - 1 = 7$ elements.

From the example, we establish a relation between all the possible intersections of the values and the subset of the set constituted by them. Thus, we can rely on the power set to guide the evaluation process. Recall that the power set of a set S – $\mathcal{P}(\mathrm{S})$ – is the set of all the subsets of S.

$$\mathcal{P}(\mathrm{S}) = \{\mathrm{X} \mid \mathrm{X} \subseteq \mathrm{S}\} \tag{14}$$

With the use of all the possible intersections, we now have a set of intervals per each sample value of $\mathcal{Y}_r^a$ and a question raises: What to do with each one of the sample values of $\mathcal{Y}_r^a$?

Table 2
DORAS – Test of all the configurations (an example)

(ψ_1, ψ_2, ψ_3)	Configuration (intersection)	result
	ψ_1	$[0.91, 2.74]$
$(\psi_1 = [0.91, 2.74]$	ψ_2	$[0.45, 1.36]$
,	ψ_3	$[9.13, 27.4]$
$\psi_2 = [0.45, 1.36]$	$\psi_1 \cap \psi_2$	$[0.91, 1.36]$
,	$\psi_1 \cap \psi_3$	$\emptyset$
$\psi_3 = [9.13, 27.4])$	$\psi_2 \cap \psi_3$	$\emptyset$
	$\psi_1 \cap \psi_2 \cap \psi_3$	$\emptyset$

Let $(\psi_1, \ldots, \psi_n) \in \prod_{i=1}^{n} \mathcal{Y}_i^a$, $n = \#\mathrm{R}_a$ be one, arbitrarily fixed, combination of individual durations and I_p implicitly defined as

$$p = \{\psi_i \,|\, i \in \mathrm{I}_p\} \in \mathcal{P}\left(\{\psi_1, \ldots, \psi_n\}\right) \setminus \emptyset \tag{15}$$

Which means that the set I_p captures the indexes of those resources contributing on a specific combination from the fixated $(\psi_1, \ldots, \psi_n)$, hence $p \in \mathcal{P}\left(\{\psi_1, \ldots, \psi_n\}\right) \setminus \emptyset$.

Example – Part 4 of 8

As a simple example, on the fourth row of Table 2, we have $p = \{\psi_1, \psi_2\}$ and $\mathrm{I}_p = \{1, 2\}$

For the remainder of this section we present some observations on the form of propositions regarding the DORAS.

Proposition 2.1

$$\forall p, q \in \mathcal{P}\left(\{\psi_1, \ldots, \psi_n\}\right) \setminus \emptyset,\, \mathrm{I}_p \cup \mathrm{I}_q = \mathrm{I}_{p \cup q} \tag{16}$$

Consider also the following function

$$\mathcal{I}\left(\mathrm{I}_p\right) = \bigcap_{v \in p} v,\, \mathrm{I}_p \in \mathrm{I}_* \tag{17}$$

where

$$\mathrm{I}_* = \{\mathrm{I}_p \,|\, p \in \mathcal{P}\left(\{\psi_1, \ldots, \psi_n\}\right) \setminus \emptyset\} \tag{18}$$

Proposition 2.2

$$\forall \mathrm{I}_p, \mathrm{I}_q \in \mathrm{I}_* \quad \mathcal{I}\left(\mathrm{I}_p \cup \mathrm{I}_q\right) = \mathcal{I}\left(\mathrm{I}_p\right) \cap \mathcal{I}\left(\mathrm{I}_q\right) \tag{19}$$

Example – Part 5 of 8

Using the newly defined function, we can rewrite the information on Table 2, about the partial intersections. In place of the fourth row, we can write

$$\mathcal{I}\left(\{1, 2\}\right) = \psi_1 \cap \psi_2 = [0.91, 1.36] \tag{20}$$

By using the function on Eq. (17), we can encapsulate all the possible partial intersections, for a given (sample) value of $\mathcal{Y}^a_{,}$ in one set.

$$\mathrm{L}'_a = \{(\mathrm{I}_p, \mathcal{I}\left(\mathrm{I}_p\right)) \,|\, \mathrm{I}_p \in \mathrm{I}_* \wedge \mathcal{I}\left(\mathrm{I}_p\right) \neq \emptyset\} \tag{21}$$

Notice that we have cutoff all the elements that represent the null partial intersections. By doing so we are intently discarding those configurations that do not yield equal durations. Thus, we apply a first filter on the partial intersections, leaving only the ones that represent non-null durations yielded by at least one resource.

Example – Part 6 of 8

Using the set representation from Eq. (21), we can replace Table 2 by the set

$$\begin{aligned} \mathrm{L'}_a = \{ & (\{1\}, [0.91, 2.74]), (\{2\}, [0.45, 1.36]), \\ & (\{1,2\}, [0.91, 1.36]), (\{3\}, [9.13, 27.4])\} \end{aligned} \tag{22}$$

We can observe that there are four partial intersections: 3 yielded by each resource alone and one yielded by two resources. Furthermore, since the resources 1 and 2 can yield together equal durations, is it useful to consider also the individual durations yielded by each one alone?

It is trivial that, on all circumstances

$$\#\mathrm{L'}_a \geqslant \#\mathrm{R}_a \tag{23}$$

since each individual duration (the ones evaluated per each resource) is always non-null. Thus, the set $\mathrm{L'}_a$ is somewhat larger than we may desire.

The goal of constructing $\mathrm{L'}_a$ with the partial intersections is to evaluate a suitable equal time interval to be a value of $\mathcal{Y}_a$; even when such equality is impossible to be absolute. Specially for this last case, it is imperative to narrow down the $\mathrm{L'}_a$ set. Next we will study the several scenarios posed by the $\mathrm{L'}_a$ set and detect non-significant elements. Finally will end with a final reasoning with the remaining intersections, leading to a suitable sample value of Y_a.

Proposition 2.3 *Given* $(\mathrm{I}_p, D_p), (\mathrm{I}_q, D_q) \in \mathrm{L'}_a$

$$D_p \cap D_q \neq \emptyset \Rightarrow (\mathrm{I}_p \cup \mathrm{I}_q, D_p \cap D_q) \in \mathrm{L'}_a \tag{24}$$

The Proposition 2.3 tells us that if two elements of $\mathrm{L'}_a$ have their durations intersections non-null, then there is in $\mathrm{L'}_a$ the element which duration is that intersection. Furthermore, the property gives the definition of such third element.

Proposition 2.4 *Given* $(\mathrm{I}_p, D_p), (\mathrm{I}_q, D_q) \in \mathrm{L'}_a$

$$\mathrm{I}_p \subseteq \mathrm{I}_q \Rightarrow D_q \subseteq D_p \tag{25}$$

The Proposition 2.3 shows how the set $\mathrm{L'}_a$ "grows" and Proposition 2.4 gives that the more resources are yielding a duration, the narrower this duration becomes. By using, until exhaustion, this last property, we are able to select those partial intersections representing durations yielded by the largest possible amount of resources. Thus, achieving the maximum consensus among the resources. In this way, the case where the absolute equality of individual durations is possible, the process will result on a single partial intersection – the one intersecting all the durations.

From the application, until exhaustion, of the Proposition 2.4, we get the set L_a.

$$\mathrm{L}_a = \mathrm{L'}_a \setminus \{(\mathrm{I}_p, D_p) \in \mathrm{L'}_a \,|\, \exists\, (\mathrm{I}_s, D_s) \in \mathrm{L'}_a \setminus \{(\mathrm{I}_p, D_p)\}, \mathrm{I}_p \subset \mathrm{I}_s\} \tag{26}$$

or, equivalently,

$$\mathrm{L}_a = \{(\mathrm{I}_p, D_p) \in \mathrm{L'}_a \,|\, \forall\, (\mathrm{I}_s, D_s) \in \mathrm{L'}_a \setminus \{(\mathrm{I}_p, D_p)\}, \mathrm{I}_p \not\subset \mathrm{I}_s\} \tag{27}$$

Example – Part 7 of 8

The L_a *for our example is obtained by removing the elements* $(\{1\}, [0.91, 2.74])$ *and* $(\{2\}, [0.45, 1.36])$ *of* $\mathrm{L'}_a$ *since* $\{1\} \subset \{1,2\}$ *and* $\{2\} \subset \{1,2\}$*, respectively. Thus,*

$$\mathrm{L}_a = \{(\{1,2\}, [0.91, 1.36]), (\{3\}, [9.13, 27.4])\} \tag{28}$$

On Fig. 1, there are graphical representations of the three typical examples of the possible $\mathrm{L'}_a$ *and* L_a *sets that can be observed.*

The calculations leading to each one of these situations will be presented on the next example.

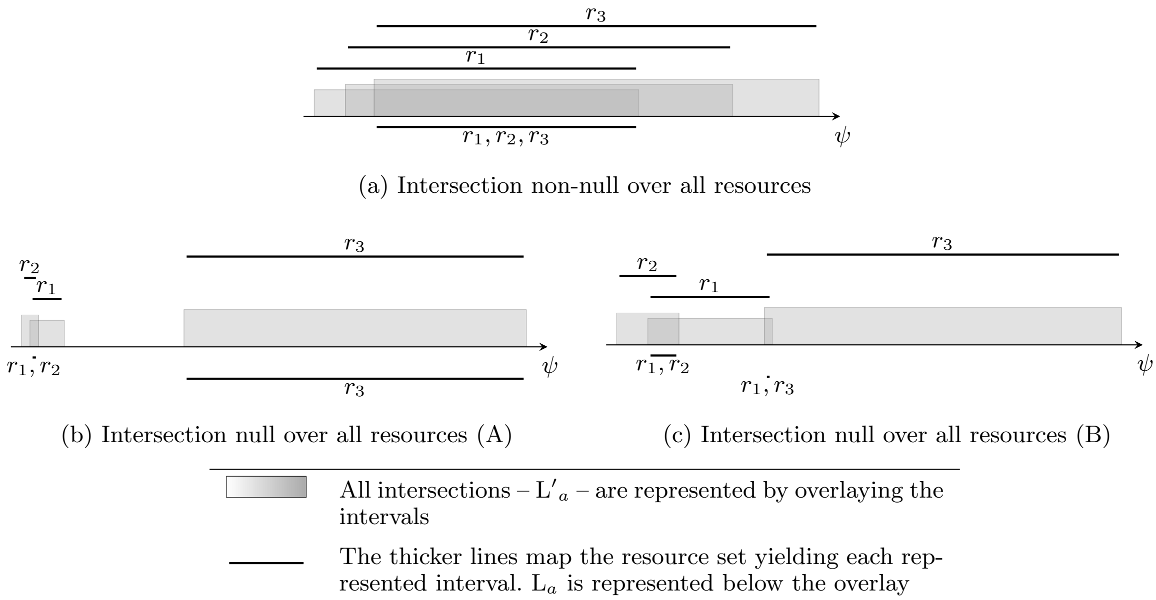

Fig. 1. DOras – Graphical representation of the three typical situations for L'_a and L_a sets

Proposition 2.5 *Let* $(I_p, D_p), (I_q, D_q) \in L_a$*:*

$$(I_p, D_p) \neq (I_q, D_q) \Rightarrow D_p \cap D_q = \emptyset \tag{29}$$

The Proposition 2.5 shows that all the durations (second component of the elements) in L_a are mutually exclusive. This aspect is important because it allows us to look at the several durations as classes. In other words, each duration interval, or class, is yielded by a specific resource subset and no other element on L_a shares any of the same duration.

Remember that we have a different L_a set per each combination of the $\mathcal{W}_r^a$ values. Thus, with each L_a we must derive a sample duration for the activity. This is typically the maximum of the individual durations. So, since the exclusion property of the durations in L_a, we can elect the class with the greater durations to be the sample value for the activity duration. This class will be named as *critical class* and referred as K. Respectively, the subset of resources securing it will be referenced as I_K.

Electing a critical class is not enough because we want to evaluate allocations for the durations. This is only possible if we use the resources subset corresponding to it. But, there is no mutual exclusion with the resources subsets in the L_a. So, it is required to analyze the three possible situations on I_K (follow with the situations on Fig. 1):

$I_K = R_a$ This case occurs when the intersection (of the individual durations) of all the activity resources is non-null (see situation a). This is the ideal scenario. By construction of L_a, there is only one element in it – the critical class. Therefore, nothing more has to be analyzed.

$I_K \cap I_i = \emptyset, \forall (I_i, D_i) \in L_a \setminus \{(I_K, K)\}$ This situation happens when all the classes have their resources sets also mutually exclusive (see situation b). This means that the critical class is supported by a subset of resources that do not support any other class. Thus, we have a exclusive set of resources yielding the critical duration. Hence, any allocation on the other resources (of the other classes) gives always durations smaller than those of the critical class. Which means that we can allocate any amount of quantity to the resources of the "inferior" classes, without influencing the activity duration.

Therefore, it is only required to evaluate allocations for the resources on I_K. All the others should always be allocated to their lower bounds, ensuring smaller cost.

$I_K \cap I_i \neq \emptyset, \forall (I_i, D_i) \in L_a \setminus \{(I_K, K)\}$ On this scenario there are some resources that support more than one class (see situation c). If none of those resources are in I_K, the same reasoning of the previous case applies. But, if some resources share more than one class, including the critical one, we must evaluate allocations for them according to the critical class, since they are involved on the evaluation of the activity duration. By doing so, we are discarding those allocations that yield the non-critical classes. This is not a problem since those do not influence the activity duration and are, actually, more costly. For all the others not involved with K, we may proceed as above.

Putting all together, to evaluate a single value for the sample $\mathcal{Y}_a - \psi$, for one given combination of $\mathcal{Y}_r^a$ values, we must construct the corresponding L_a and in it, select the critical class K and the supporting resources subset $\mathrm{I_K}$. In fact, each ψ will be the element of L_a that has the critical class.

Example – Part 8 of 8

In this final example, we will summarize the allocation process by giving three complete examples, representative of each of the notable cases above; on the same context of this running examples. Recall the chosen work content samples and the Eq. (10).

First Scenario (situation a) *By picking the following samples of work content*

$$\omega_1 = 10 \qquad \omega_2 = 11.93 \qquad \omega_3 = 13.7 \tag{30}$$

it follows

$$\psi_1 = [6.67, 20] \qquad \psi_2 = [7.95, 23.86] \qquad \psi_3 = [9.13, 27.4] \tag{31}$$

From which,

$$\begin{aligned} \mathrm{L'}_a = \{ & (\{1\}, \psi_1), (\{2\}, \psi_2), (\{3\}, \psi_3), (\{1,2\}, [7.95, 20]), \\ & (\{1,3\}, [9.13, 20]), (\{2,3\}, [9.13, 23.86]), \\ & (\{1,2,3\}, [9.13, 20])\} \end{aligned} \tag{32}$$

and

$$\mathrm{L}_a = \{(\{1,2,3\}, [9.13, 20])\} \tag{33}$$

Thus, $\mathrm{K} = [9.13, 20]$ *and* $\mathrm{I_K} = \{1,2,3\}$*. On Table 3 we have some allocation vectors from these data.*

Table 3
DORAS – Allocation example (a)

$d \in \mathrm{K}$	$x_1^a = \omega_1/d$	$x_2^a = \omega_2/d$	$x_3^a = \omega_3/d$	
9.13	1.095	1.307	1.500	$\omega_1 = 10.0$
11.848	0.844	1.007	1.156	$\omega_2 = 11.93$
14.565	0.687	0.819	0.940	$\omega_3 = 13.7$
17.283	0.579	0.690	0.792	$\mathrm{K} = [9.13, 20]$
20	0.500	0.597	0.685	$\mathrm{I_K} = \{1,2,3\}$

On this scenario it is possible to evaluate allocation vectors with no wasteful durations.

Second Scenario (situation b) *This is the example that guided us until now.*

$$\omega_1 = 1.37 \qquad \omega_2 = 0.68 \qquad \omega_3 = 13.7 \tag{34}$$

it goes

$$\psi_1 = [0.91, 2.74] \qquad \psi_2 = [0.46, 1.37] \qquad \psi_3 = [9.13, 27.4] \tag{35}$$

From which,

$$\mathrm{L'}_a = \{(\{1\}, \psi_1), (\{2\}, \psi_2), (\{3\}, \psi_3), (\{1,2\}, [0.91, 1.37])\} \tag{36}$$

and

$$\mathrm{L}_a = \{(\{1,2\}, [0.91, 1.37]), (\{3\}, [9.13, 27.4])\} \tag{37}$$

Thus, $\mathrm{K} = [9.13, 27.4]$ *and* $\mathrm{I_K} = \{3\}$*. On Table 4 we have some allocation vectors from these data. In this case, there is only one resource responsible for the activity duration.*

Table 4
DORAS – Allocation example (b)

$d \in \mathrm{K}$	$x_1^a = l_1^a$	$x_2^a = l_2^a$	$x_3^a = \omega_3/d$
9.13	0.5	0.5	1.500
13.695	0.5	0.5	1.000
18.261	0.5	0.5	0.750
22.826	0.5	0.5	0.600
27.4	0.5	0.5	0.500

$\omega_1 = 1.37$
$\omega_2 = 0.68$
$\omega_3 = 13.7$
$\mathrm{K} = [9.13, 27.4]$
$\mathrm{I_K} = \{3\}$

Third Scenario (situation c) *This time, by choosing*

$$\omega_1 = 4.77 \qquad \omega_2 = 2.38 \qquad \omega_3 = 13.7 \tag{38}$$

we have

$$\psi_1 = [3.18, 9.54] \qquad \psi_2 = [1.59, 4.77] \qquad \psi_1 = [9.13, 27.4] \tag{39}$$

From which,

$$\begin{aligned} \mathrm{L}'_a = \{ & (\{1\}, \psi_1), (\{2\}, \psi_2), (\{3\}, \psi_3), (\{1,2\}, [3.18, 4.77]), \\ & (\{1,3\}, [9.13, 9.54])\} \end{aligned} \tag{40}$$

and

$$\mathrm{L}_a = \{(\{1,2\}, [3.18, 4.77]), (\{1,3\}, [9.13, 9.54])\} \tag{41}$$

Thus, $\mathrm{K} = [9.13, 9.54]$ *and* $\mathrm{I_K} = \{1, 3\}$. *On Table 5 we have some allocation vectors from these data. Here, the allocations for the resource 1 are being guided by the critical class only, despite also supporting other class.*

Table 5
DORAS – Allocation example (c)

$d \in \mathrm{K}$	$x_1^a = \omega_1/d$	$x_2^a = l_2^a$	$x_3^a = \omega_3/d$
9.13	0.522	0.5	1.500
9.231	0.516	0.5	1.484
9.333	0.511	0.5	1.468
9.434	0.505	0.5	1.452
9.54	0.500	0.5	1.436

$\omega_1 = 4.77$
$\omega_2 = 2.38$
$\omega_3 = 13.7$
$\mathrm{K} = [9.13, 9.54]$
$\mathrm{I_K} = \{1, 3\}$

3 Discussion and Conclusion

The DORAS deals well with an arbitrary number of resources. The all process is oblivious of the number of resources. But, since it works down with all possible situations on a given context, that is also a drawback of DORAS – complexity.

To give an idea of the complexity of the DORAS, let n be the number of resources of an activity and p the number of sample values for the $\mathcal{W}_r^a$. The process begins by sampling the work contents and then walks trough all the combinations of sample values among the resources. Thus, p^n combinations. Then it computes all the possible intersections guided by the power set, giving $2^n - 1$ intersections per combination. Thus, we have $p^n(2^n - 1)$ elements already treated by DORAS. Then, the process continues by filtering redundant elements. This is done for each of possible elements and consists on an operation where it may be required to test nearly all elements against one another! Notice the real implications of this amount of operations when they must be carried for all the activities on a project, several times during the optimization cycles.

You may think that if DORAS was simplified by simply discard the null absolute intersections, the complexity would fall down. It sure would help. But, there is another problem with DORAS. And this one is not easily avoided.

Each resource supporting the critical class, is doing it using a portion of its allocation interval. But, the resources do not contribute with the same ranges. The (quantity) allocations are therefore very conditioned by that fact. If, at implementation phase, we make a uniform partition of the critical duration (such as we have done with the last examples) we are incurring in a major bias on the evaluated quantities: some contributing resources do so with nearly equal quantity amounts while others are more sparse. This is dangerous because hides from the optimization process precious allocations that might result in better final costs. Thus, instead of an uniform partitioning we must endorse a partitioning system sensible to the contribution gap between the contributing resources. And this, alone, is very complex.

Despite being so complex, `DOras`, together with the ease of generalization, allows the identification of situations when some resources may be redistributed. We consider the `DOras` a good exercise on the study of the activities duration with multiple resources because it allows us a better insight about the allocation implications on activity duration. Yet, we choose to diverge our search for another and much simpler and computational friendly strategy, the `WBras` (*Waste Balance Resource Allocation Strategy*) presented last year in `EngOpt2008` [4].

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A Proofs of the propositions

Proof of Proposition 2.1

$$\begin{aligned} x \in \mathrm{I}_p \cup \mathrm{I}_q &\Leftrightarrow x \in \mathrm{I}_p \vee x \in \mathrm{I}_q && \langle \text{DEF. } \cup \rangle \\ &\Leftrightarrow \psi_x \in \mathrm{I}_p \vee \psi_x \in \mathrm{I}_q && \langle \text{DEF. } \mathrm{I}_p \text{ AND } \mathrm{I}_q \rangle \\ &\Leftrightarrow \psi_x \in p \cup q && \langle \text{DEF. } \cup \rangle \\ &\Leftrightarrow x \in \mathrm{I}_{p\cup q} && \langle \text{DEF. } \mathrm{I}_p \text{ AND } \mathrm{I}_q \rangle \end{aligned}$$

□

Proof of Proposition 2.2

$$\begin{aligned} \mathcal{I}(\mathrm{I}_p \cup \mathrm{I}_q) &= \mathcal{I}(\mathrm{I}_{p\cup q}) && \langle \text{PROPOSITION } 2.1 \rangle \\ &= \bigcap_{v \in p \cup q} v && \langle \text{DEF. } \mathcal{I}(-) \text{ AT EQ. } (17) \rangle \\ &= \bigcap_{v' \in p} v' \cap \bigcap_{v'' \in q} v'' && \langle \text{DEF. } \cap \text{ AND DEF. } \cup \rangle \\ &= \mathcal{I}(\mathrm{I}_p) \cap \mathcal{I}(\mathrm{I}_q) && \langle \text{DEF. } \mathcal{I}(-) \rangle \end{aligned}$$

□

Proof of Proposition 2.3

We want to prove that $(\mathrm{I}_p \cup \mathrm{I}_q, D_p \cap D_q) \in \mathrm{L'}_a$. Since $D_p \cup D_q \neq \emptyset$ (by hypothesis), by definition of $\mathrm{L'}_a$, that is equivalent to prove

$$\mathcal{I}(\mathrm{I}_p \cup \mathrm{I}_q) = D_p \cap D_q$$

$$\begin{aligned} \mathcal{I}(\mathrm{I}_p \cup \mathrm{I}_q) &= \mathcal{I}(\mathrm{I}_p) \cap \mathcal{I}(\mathrm{I}_q) && \langle \text{Proposition 2.2} \rangle \\ &= D_p \cap D_q && \langle \text{hyp. } (\mathrm{I}_p, D_p), (\mathrm{I}_q, D_q) \in \mathrm{L'}_a \rangle \end{aligned}$$

□

Proof of Proposition 2.4

From the hypothesis we get:

(1) $(\mathrm{I}_p, D_p) \in \mathrm{L'}_a \Rightarrow D_p = \mathcal{I}(\mathrm{I}_p) = \bigcap_{v \in p} v \neq \emptyset$
(2) $(\mathrm{I}_q, D_q) \in \mathrm{L'}_a \Rightarrow D_q = \mathcal{I}(\mathrm{I}_q) = \bigcap_{v \in q} v \neq \emptyset$

Also, $\mathrm{I}_p \subseteq \mathrm{I}_q \Leftrightarrow \mathrm{I}_p = \mathrm{I}_q \vee \mathrm{I}_p \subsetneq \mathrm{I}_q$. So we must cope with two cases.

Case $\mathrm{I}_p = \mathrm{I}_q$ then $D_q = D_p$, trivially by definition of $\mathcal{I}(-)$.

Case $\mathrm{I}_p \subsetneq \mathrm{I}_q$ then $\mathrm{I}_q = \mathrm{I}_p \cup X$ where

- $X \in \mathrm{I}_*$ $\langle$trivial$\rangle$
- $X \neq \mathrm{I}_p$ $\langle \mathrm{I}_q \neq \mathrm{I}_p$, in this case$\rangle$
- $X \neq \emptyset$ $\langle \mathrm{I}_q \neq \mathrm{I}_p$, in this case$\rangle$

Because $X \in \mathrm{I}_*$, let us assume

$$X = \mathrm{I}_s,\ s \in \mathcal{P}(\{\psi_1, \ldots, \psi_n\})$$

So,

$$\begin{aligned} \mathrm{I}_q &= \mathrm{I}_p \cup X \\ &= \mathrm{I}_p \cup \mathrm{I}_s && \langle \text{Assumed } X = \mathrm{I}_s \rangle \\ &= \mathrm{I}_{p \cup s} && \langle \text{Proposition 2.1} \rangle \end{aligned}$$

Thus,

$$\begin{aligned} D_q &= \mathcal{I}(\mathrm{I}_q) && \langle \text{item 2} \rangle \\ &= \mathcal{I}(\mathrm{I}_{p \cup s}) && \langle \mathrm{I}_q = \mathrm{I}_{p \cup s} \rangle \\ &= \mathcal{I}(\mathrm{I}_p) \cap \mathcal{I}(\mathrm{I}_s) && \langle \text{Proposition 2.2} \rangle \\ &= D_p \cap \mathcal{I}(\mathrm{I}_s) && \langle \text{item 1} \rangle \\ &\subsetneq D_p && \langle \mathcal{I}(\mathrm{I}_s) \neq \emptyset \rangle \end{aligned}$$

Notice $\mathcal{I}(\mathrm{I}_s) \neq \emptyset$ otherwise $D_q = \emptyset$, contradicting the hypothesis of $(\mathrm{I}_q, D_q) \in \mathrm{L'}_a$.

Therefore, putting the two cases together,

$$D_q \subseteq D_p$$

□

Proof of Proposition 2.5

Suppose, by absurd, that there are two elements (I_p, D_p) and (I_q, D_q) of L_a such that $D_p \cap D_q \neq \emptyset$.

Since, by construction of L_a, all elements belonging to L_a also belong to $\mathrm{L'}_a$, we may use the Proposition 2.3. Then,

$$(\mathrm{I}_p \cup \mathrm{I}_q)\, D_p \cap D_q \in \mathrm{L'}_a$$

But, for example, $\mathrm{I}_p \subset \mathrm{I}_p \cup \mathrm{I}_q$. Thus, by construction of L_a, $(\mathrm{I}_p, D_p) \notin \mathrm{L'}_a$. Which is **Absurd**.

$\therefore D_p \cap D_q = \emptyset$

□