Basic Approximations to an Adaptive Resource Allocation Technique to Stochastic Multimodal Projects

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Abstract

This paper presents three basic approximations developed to solve the Adaptive Stochastic Multimodal Resource Allocation Problem. Two of them are based on the DP model introduced in earlier papers ([23], [24]). The other one uses NLP to solve this problem. The approximations developed consist in considering the Work Content of some or all the activities of the project as represented by their mean values. These approximations were applied to a set of examples, and results were obtained and commented. As expected, running times were reduced, compared to the original model, but the total cost was underestimated, due to the use of means instead of the complete distribution.

Key Words: Activity Networks, Resource Allocation, Dynamic Programming

1   Introduction and Background

After defining a dynamic programming (DP) model to be applied to the adaptive resource allocation in stochastic multimodal project networks (see [23]), and after

having implemented and tested the model with a set of networks (see [24]), we proceeded to develop basic approximations to this model, due to the complexity of the problem. These approximations were able to reduce computation time considerably, with still good results.

Before introducing the approximations, we are going to briefly define the problem. Given a multimodal activity network[1], with a stochastic work content (W_a), we want to decide the amount of resource to apply to each activity (x_a), so that the total cost is minimized. This cost includes the resource cost and the delay cost. The duration of an activity depends on its work content and on the amount of resource allocated to it. To evaluate the delay cost, a due date must be specified (T), as well as the unit cost per period tardy (c_L). To the best of our knowledge this problem has never been treated before. Contributions to the classical 'resource constrained project scheduling problem' (RCPSP) and its variants are numerous; the interested reader may wish to consult the two most recent books on the subject by Demeulemeester and Herroelen (2002) [10] and Neumann, Schwindt, and Zimmermann (2002) [20], and the references cited therein, to gain a complete picture of developments in that aspect of project scheduling.

We imposed the following assumptions:

- The work content of each activity is a random variable (*r.v.*) exponentially distributed[2].

- The amount of resource is specified within a lower and an upper bound $[l, u]$.

- The availability of the resource is unlimited, so it doesn't impose any limitations to the problem.

The model developed to solve this problem will be reviewed, using a simple example with only three activities (see figure 1).

The due date of the project is $T = 9$, and the unit cost per period tardy is $c_L = 4$. The resource allocation to each activity is denoted by x_i for $i = 1, 2, 3$; with lower limit $l_i = 0.5$ and upper limit $u_i = 1.5$ for all i. The x_i's are the *decision variables* of this problem. The parameters $\{\lambda_i\}$ of the distributions of the work content of the activities are as shown in table 1.

Table 1: Parameters for the simple example

Activity i :	1	2	3
λ_i :	0.4	0.16	0.1

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[1]That is, each activity can be performed in any number of levels of resource intensity applied to it, with resulting shorter or longer duration.

[2]This assumption was done for simplicity of exposition and computing.
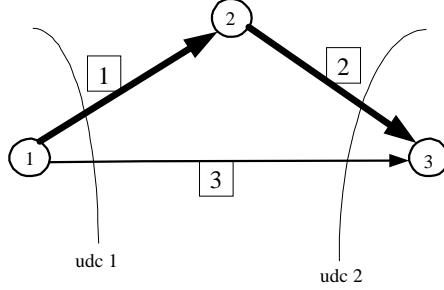
Figure 1: Example network with its uniformly directed cutsets.

At any point in time the manager must cope with a subset of activities that lie on a uniformly directed cutset (udc)[3]. In this simple example there are only two udc's: $C_1 = \{1, 3\}$ and $C_2 = \{2, 3\}$. To be sure, at the outset the project manager is concerned with activities 1 and 3, which lie on C_1. Then, depending on the progress in these two activities, he may eventually be concerned with activities 2 and 3, which lie on C_2. If the resource allocation to activity 3 is (temporarily) fixed at, say, $\hat{x}_3$, the problem reduces to the optimal determination of the resource allocation to activities 1 and 2, which can be readily resolved by standard DP recursion. The set of "fixed" activities is denoted by $\mathcal{F}$; in this example $\mathcal{F} = \{3\}$. Finally, searching over the values of x_3 with repeated optimization at each value would yield the (unconditional) optimum allocation to all three activities. In general, our procedure determines the udc's of the network (which define the stages of the DP iterative scheme), and the cutset intersection index (cii), which represents the variables to be (temporarily) fixed (see [23] for details).

The expected resource cost of the fixed variables is denoted by rcf, which in this case, is the expected cost of activity 3

$$rcf = \hat{x}_3 \cdot \mathcal{E}\left(W_3\right) = \frac{\hat{x}_3}{0.1}, \tag{1}$$

where W_3 is the work content of activity 3, $\mathcal{E}\left(W_3\right)$ denotes its expected value, and $\hat{x}_3$ the amount of resource allocated to it. Reverse numbering of the DP stages yields

$$f_1(t_2 \,|\, \mathcal{F} = \{3\}) = rcf + \min_{x_2 \in [0.5, 1.5]} \mathcal{E}\left\{x_2 W_2 + 4\mathcal{E}\left(U\right)\right\}, \tag{2}$$

where

$$U = \max\{0, \Upsilon_3 - T\}, \tag{3}$$

and

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[3]A udc represent a set of possible active activities, during the life of the project.

$$\Upsilon_3 = \max\{t_2 + \frac{W_2}{x_2}, \frac{W_3}{\hat{x}_3}\}. \tag{4}$$

The second and last stage would be defined as follows:

$$f_2(t_1 = 0 \,|\, \mathcal{F} = \{3\}) = \min_{x_1 \in [0.5, 1.5]} \mathcal{E}\{x_1 W_1 + \mathcal{E}[f_2(\Upsilon_2)]\} \tag{5}$$

where

$$\Upsilon_2 = \frac{W_1}{x_1}. \tag{6}$$

The solution for this network, obtained in 0.22 seconds[4], is:

$$\{x_1^*, x_3^*\} = \{1.25, 1.5\} \tag{7}$$

with an expected cost of 35.97.

The optimal value of x_2 depends on the state of node 2, when it is reached, and can be obtained by the previously developed optimal policy for stage 1, as defined in equation (2). The time necessary to get results in this example is considerably small, but for bigger networks, this time increases exponentially, taking hours and even days to achieve[5].

This model was implemented in Matlab. The pseudo-code can be accessed on the internet[6], or upon request by e-mail[7].Details of the development of this application can be seen in our previous paper ([24]).

2   The Approximations Developed

In this section we will introduce the basic approximations developed to solve this problem. Two of them still use DP, and one uses non linear programming (NLP). These approximations were developed considering that the Work Content of some or all the activities of the project is represented by the mean value. The approximations were applied to a set of examples, and results were obtained and will be commented. As expected, running times were reduced, compared to the original model, but the total cost was underestimated, due to the use of means instead of the complete distribution ([13]).

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[4]The computer used to do the experimentations was a Pentium III, 650 MHz, 128 MB.

[5]See the order of complexity of this problem in our previous paper ([24]).

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2.1 Approximation 1

This approximation is based on the DP model introduced ([23], [24], [25]), but considers the Work Contents of the fixed activities (set $\mathcal{F}$) as represented by their mean values. For the simple network, only W_3 would be approximated to its mean ($\frac{1}{\lambda} = \frac{1}{0.1} = 10$).

The approximated solution was obtained in 0.19 seconds, and it is stated in (8).

$$\{x_1^*, x_3^*\} = \{1.25, 1.0\} \tag{8}$$

with an expected cost of 27.65.

2.2 Approximation 2

This approximation is based on the DP model introduced and represents all random variables by their mean values. These variables are the Work Contents of all the activities. In the simple example above, we would have:

$$\{W_1, W_2, W_3\} = \{2.50, 6.25, 10.00\}. \tag{9}$$

The solution for the simple network, was obtained in 0.12 seconds:

$$\{x_1^*, x_3^*\} = \{1.25, 1.0\} \tag{10}$$

with an expected cost of 21.81.

2.3 Approximation 3

This approximation uses NLP and considers all random variables as represented by their mean values. These variables are the Work Contents of the all the activities. In the simple example above, the work content will be as shown in (9).

The model was developed in the Excel Solver. The goal is to minimize the total cost of the project, which includes the resource cost and the delay cost. The non-linear function that represents the total cost, for the example project, can be seen in (11).

$$F(x) = x_1 W_1 + x_2 W_2 + x_3 W_3 + c_L \max(0, t_3 - T). \tag{11}$$

The parameters introduced were the values of λ, for all the activities, and the correspondent work content was evaluated using expression (12).

$$W = \frac{1}{\lambda}. \tag{12}$$

It was also introduced the value of $T = 9$ and $C_L = 4$. The goal is to determine the values of x, that minimize the function $F(x)$.

It is also necessary to know the value of $t3$, the time of realization of the last node of the network. This value can be obtained using the expression (13).

$$t_3 = \max(t_2 + \frac{W_2}{x_2}, t_1 + \frac{W_3}{x_3}), \tag{13}$$

where $t_2 = t_1 + \frac{W_1}{x_1}$ and $t_1 = 0$.

It is also necessary to include restrictions for the x limits, of the type $0.5 \leq x \leq 1.5$.

Then, the minimization of the objective function was done, and the result, (14), was obtained in 0.1 seconds.

$$\{x_1^*, x_2^*, x_3^*\} = \{0.8, 1.06, 1.11\} \tag{14}$$

with an expected cost of 19.79.

The Excel sheet that allowed to solve this problem, can be seen in figure 2.

Activity a	Lambda a	Wa		xa	time ti		F(x)=	19,79
1	0,4	2,5	x1=	0,8	0 : t1=0			
2	0,16	6,25	x2=	1,06	3,113 : t2=t1+W1/x1			
3	0,1	10	x3=	1,11	9,021 : t3=max(t1+W3/x3;t2+W2/x2)			
			delay u=	0,02	u=t3-T	F(x)=x1W1+x2W2+x3W3 +d*max(0;u)		
			T=	9				
			d=	4				

Figure 2: NLP in Excel (Simple example)

3   Examples and Results

The initial model and subsequent approximations were tested on a set of four projects that range in size from 5 to 18 activities. These examples are shown in appendix A, and the solutions obtained in appendix B. The solution times varied from a few *seconds* to five *days* on a Pentium III, 650 MHz, 128 MB. The program output, for the DP case, indicates the "optimal" cost and the "best" values for the variables that emanate from node 1, as well as the "best" values for the fixed variables. The values of the remaining decision variables depend on the state of the project, at the time of the decision, and can be determined from the optimal policies developed for the

corresponding stage. The words *optimal* and *best* are put between quotation marks because they are not the *absolute* optima and best due to the discretization of the work content and the times of node realizations. Finer meshes may result in improved optima, at the price of (greatly) increased computational effort. In the NLP case, the discretization error does not exist, but the values obtained are not optimal, due to the approximations.

There are two important remarks:

- Network 3 has only one more activity than network 2, but the time necessary to get results (in the DP Model case) increases from 30 minutes to 23 hours. This was due to the increased number of fixed activities (from six to eight). Thus the number of combinations generated for the fixed variables increased from $3^6 = 729$ to $3^8 = 6561$.

- To be able to obtain the solution for network 4, it was necessary to reduce the number of points considered, for the resource allocation of the fixed variables, to two[8]. Even after this reduction, the results were obtained after 5 days, using the straight DP model.

4   Conclusions and Future Research

After these set of experiments we can see that for the straight DP model, and for the larger networks, the time necessary to get results is prohibitive. In theory, this is an exact model. Armed with a powerful computer, we would be able to get real optimal values. As this is not possible, we tried to develop simple approximations, to start with, to improve computational time. And this was accomplished. As more variables are approximated, the computational time is reduced. With NLP, the reduction is even bigger.

The values obtained using the DP model can be seen as upper limits to the optimum. This is due to the discretizations needed.

As we do approximations, the expected value of the cost tends to be lower, because, when we use means instead of the complete distribution, to represent the work contents, the true conclusion dates are underestimated ([13]). That explains why the values tend to decrease, as more variables are approximated.

Theoretically, approximations 2 and 3 should give the same results. The differences observed are due to the discretizations needed in the DP model.

But rather then looking at the expected value of the cost, it is also important to look at the values of the levels of resources (x's) proposed by the model and the approximations. There are cases in which these values don't change much. Since these values are stable, it is a good recomendation to use them, when implementing the

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[8]For the other networks, the quantities of resources for the fixed variables were discretized using three points.

project. The others may not be so important, to the final result. In approximation 3, the x values are different from the others, because in NLP, it is not necessary to use discretization, as in DP.

As future research we plan to keep improving results, times and searching for better approximations to the optimum, namelly:

- Forfeit some 'managerial flexibility' in the adaptive optimization by 'aggregating' activities. The very act of 'aggregation' combines two or more activities into a larger 'aggregate activity', which will necessarily delete nodes in the original network. All aggregated activities shall be treated as a single activity to which a resource allocation shall be made. This robs the manager of the flexibility of varying the allocation to individual activities according to progress to date, which was stated in the realization of the (deleted) nodes.

- Forfeit the generality of the DP approach in favor of the specialized treatment of exponentially distributed $r.v.$'s which lead to the interpretation of the project as a continuous time Markov chain. Such analysis may provide upper bound on the expected cost of the project, which may be useful in the budgeting/bidding process.

Finally, the DP approach is very demanding computationally, in whichever form it is used. So we are compelled to try to apply other compu-search approaches to this problem. These approaches will involve the use of various techniques such as Monte Carlo Simulation combined with CPM evaluation, and a global optimization technique based on the 'Electromagnetism Algorithm' designed by Birbil and Fang[5].

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A Example Networks and Parameters

A.1 Network 1
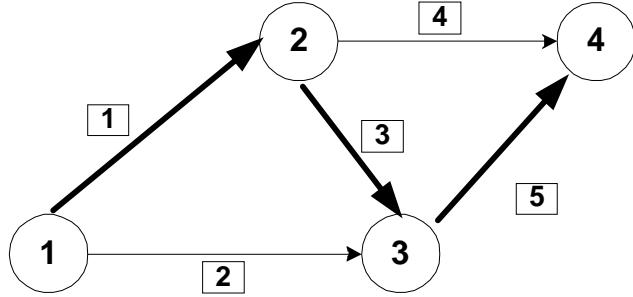
Figure 3: Network 1

The first network has 5 activities (see figure 3). $T = 120$ and $c_L = 8$. In table 2 are the remaining parameters.

Table 2: Parameters for network 1

Activity	1	2	3	4	5
Origin	1	1	2	2	3
Target	2	3	3	4	4
λ	0.02	0.03	0.04	0.024	0.025
$\mathbf{x}_{\min}$	0.5	0.5	0.5	0.5	0.5
$\mathbf{x}_{\max}$	1.5	1.5	1.5	1.5	1.5

A.2 Network 2

Network 2 (see figure 4) is of larger dimension (11 activities). For this network, $T = 28$ and $c_L = 8$. The remaining parameters are presented in table 3.

Table 3: Parameters for network 2

Activity	1	2	3	4	5	6	7	8	9	10	11
Origin	1	1	1	2	3	2	3	4	3	5	4
Target	2	3	4	3	4	5	5	5	6	6	6
λ	0.1	0.09	0.4	0.2	0.3	0.08	0.4	0.2	0.1	0.3	0.3
$\mathbf{x}_{\min}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\mathbf{x}_{\max}$	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
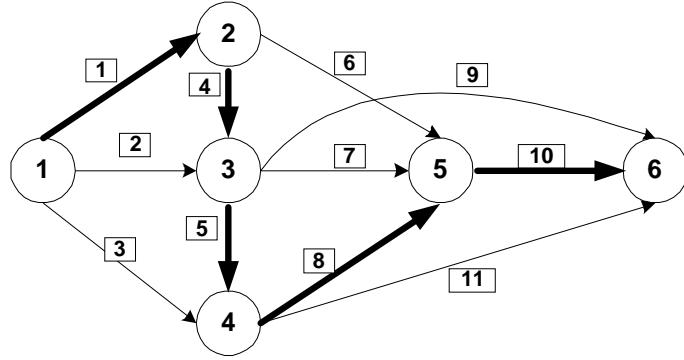
Figure 4: Network 2

A.3 Network 3
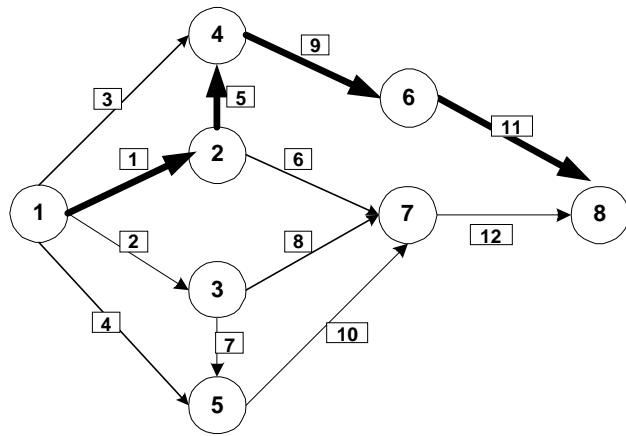
Figure 5: Network 3

Network 3 has one more activity than the last one (see figure 5), and different topology. $T = 47$ and $c_L = 4$. The remaining parameters are presented in table 4.

Table 4: Parameters for network 3

Activity	1	2	3	4	5	6	7	8	9	10	11	12
Origin	1	1	1	1	2	2	3	3	4	5	6	7
Target	2	3	4	5	4	7	5	7	6	7	8	8
λ	0.1	0.09	0.08	0.1	0.09	0.08	0.1	0.09	0.08	0.1	0.09	0.1
$\mathbf{x}_{min}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\mathbf{x}_{max}$	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

A.4 Network 4
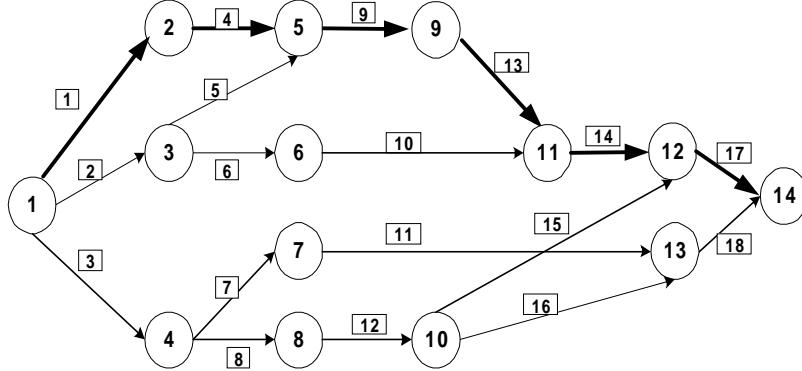
Figure 6: Network 4

To finish this set of tests, we used the network of figure 6, which has a considerable size compared to the previous ones. Here, $T = 110$ and $c_L = 10$. The remaining parameters are presented in table 5.

Table 5: Parameters for network 4

Activity	1	2	3	4	5	6	7	8	9
Origin	1	1	1	2	3	3	4	4	5
Target	2	3	4	5	5	6	7	8	9
λ	0.06	0.04	0.1	0.07	0.08	0.04	0.08	0.2	0.07
x_{min}	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
x_{max}	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

Activity	10	11	12	13	14	15	16	17	18
Origin	6	7	8	9	11	10	10	12	13
Target	11	13	10	11	12	12	13	14	14
λ	0.05	0.08	0.07	0.09	0.09	0.05	0.09	0.04	0.06
x_{min}	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
x_{max}	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

B Solutions

B.1 Network 1

Table 6: Results for network 1

Network 1	DP Model	Approx. 1	Approx. 2	Approx. 3
$\mathbf{x}_1$	1.0	1.0	1.25	1.03
$\mathbf{x}_2$	1.0	1.0	0.5	0.51
$\mathbf{x}_3$				0.87
$\mathbf{x}_4$	1.5	1.0	0.5	0.59
$\mathbf{x}_5$				0.96
$EV(Cost)$	304.62	279.14	148.79	152.82
$Run\ Time$	9.6 sec.	8.4 sec.	6 sec.	1 sec.

.

B.2 Network 2

Table 7: Results for network 2

Network 2	DP Model	Approx. 1	Approx. 2	Approx. 3
$\mathbf{x}_1$	1.25	1.25	0.75	1.15
$\mathbf{x}_2$	1.0	1.0	1.0	0.8
$\mathbf{x}_3$	0.5	0.5	0.5	0.5
$\mathbf{x}_4$				0.95
$\mathbf{x}_5$				1.16
$\mathbf{x}_6$	1.0	0.5	1.0	0.82
$\mathbf{x}_7$	0.5	0.5	0.5	0.5
$\mathbf{x}_8$				0.72
$\mathbf{x}_9$	1.0	0.5	1.0	0.9
$\mathbf{x}_{10}$				0.82
$\mathbf{x}_{11}$	1.0	0.5	0.5	0.5
$EV(Cost)$	106.76	75.04	58.94	58.75
$Run\ Time$	30 min.	13 min.	6 min.	1 sec.

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B.3   Network 3

Table 8: Results for network 3

Network 3	DP Model	Approx. 1	Approx. 2	Approx. 3
$\mathbf{x}_1$	1.25	1.25	0.75	0.93
$\mathbf{x}_2$	1.0	0.5	1.0	0.81
$\mathbf{x}_3$	1.0	1.0	0.5	0.55
$\mathbf{x}_4$	0.5	0.5	0.5	0.5
$\mathbf{x}_5$				0.93
$\mathbf{x}_6$	1.0	0.5	0.5	0.5
$\mathbf{x}_7$	1.0	0.5	1.0	0.82
$\mathbf{x}_8$	0.5	0.5	0.5	0.5
$\mathbf{x}_9$				0.97
$\mathbf{x}_{10}$	1.0	0.5	1.0	0.82
$\mathbf{x}_{11}$				0.97
$\mathbf{x}_{12}$	1.5	1.0	1.0	1.15
$EV(Cost)$	182.91	117.92	103.26	91.53
$Run\ Time$	23 h.	7 h.	3 h.	1 sec.

.

B.4 Network 4

Table 9: Results for network 4

Network 4	DP Model	Approx. 1	Approx. 2	Approx. 3
$\mathbf{x}_1$	0.75	1.25	0.5	0.69
$\mathbf{x}_2$	1.25	1.25	0.75	1.3
$\mathbf{x}_3$	1.25	0.75	0.75	0.5
$\mathbf{x}_4$				0.67
$\mathbf{x}_5$	0.75	0.75	0.75	0.5
$\mathbf{x}_6$	1.25	1.25	0.75	1.29
$\mathbf{x}_7$	0.75	0.75	0.75	0.5
$\mathbf{x}_8$	1.25	0.75	0.75	0.5
$\mathbf{x}_9$				0.66
$\mathbf{x}_{10}$	1.25	0.75	0.75	1.14
$\mathbf{x}_{11}$	1.25	0.75	0.75	0.5
$\mathbf{x}_{12}$	0.75	0.75	0.75	0.59
$\mathbf{x}_{13}$				0.73
$\mathbf{x}_{14}$				1.02
$\mathbf{x}_{15}$	0.75	0.75	0.75	0.58
$\mathbf{x}_{16}$	1.25	0.75	0.75	0.55
$\mathbf{x}_{17}$				1.5
$\mathbf{x}_{18}$	0.75	0.75	0.75	0.56
$EV(Cost)$	339.07	271.38	210.19	144.07
$Run\ Time$	5 days	9 h.	2 h.	1 sec.

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