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PROV Exponential decision method

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- How to discover the most adequate solution for a decision problem when we have more than two alternatives of choice?

Research question

- To develop a complete multicriteria decision method using the exponential normalization to express the decision-maker knowledge, preferences and purposes to attain comprehensible results and to discover the most adequate solution or set of solutions for a problem.

Research objective

- The development of the PROV Exponential Decision Method
  - (decision-maker Preferences Ranking and Options Value based on the linear and on the exponential normalization).

Research result

- Multicriteria decision methods are applied to find the most appropriate solution for a specific problem or to attain a certain goal.
- Among the most known multicriteria decision methods adressing the decision-maker preferences are the:
  - AHP – Analytic Hierarchy Process
  - ELECTRE – Élimination et Choice Traduisant la Réalité
  - PROMETHEE – Preference ranking organization method for enrichment evaluations

Research context

- ELECTRE and PROMETHEE are outranking methods which use reference threshold to encompass the limitations of the linear normalization.
- AHP uses a nine-points preference scale to rank the options and their criteria through paired-wise comparisons.
- There isn't any complete multicriteria decision method using the exponential normalization and using simultaneously the concepts of preference, indifference and nefarious values modeled on a graphical representation.

Research context

Research context

- The PROV Exponential Decision Method uses the concepts of preference, indifference and nefarious thresholds on a graphical representation where we can observe the relative position of every option on two lines, the linear and the exponential line, to determine the values best representing the decision-maker thoughts and intentions.

- The linear function expresses that increments of the same size have equal importance.

Linear function

- The exponential function expresses the actual value attributed by the decision-maker.
  - As some milestones are attained the importance attributed to greater values may decrease, since some value of satisfaction has been obtained.
  - It also lets the decision-maker to express the interval of values at which he considers the options indifferent among each other.
  - It’s also possible to express the decrease of preference if, at a determined level, the continuous growth becomes nefarious.

Exponential function

Application scope

- Whenever we are on the presence of more than two options of choice the PROV Exponential Decision Method can be applied.
  - It can be particularly important on:
    - investment decisions;
    - product portfolio assessment;
    - evaluation of intangible assets and intellectual capital;
    - policy appraisal and public funding, concerning social interventions, the environment, quality control decisions and health and safety issues;
    - products and equipments acquisition and implementation.

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- 1st Establish the overall objective to be achieved;
- 2nd Enounce the main requisites that a solution for the problem will have to accomplish;
- 3rd Identify reasonably practicable alternatives of solution (our options);
- 4th Enounce all the options relevant criteria to be taken into account during the analysis;

Application procedure

Application procedure

- 5th Identify the attributes for each option and establish a matrix with those attributes.
  - If we have qualitative criteria, we should establish *Likert scales* to make them quantifiable (every criterion may have their own independent reference scale; *a posteriori*, they will be normalized in a scale between 0 and 1);

	Options				
Criteria	o_1	o_2	o_3	o_4	o_m
c_1	x_{11}	x_{21}	x_{31}	x_{41}	x_{m1}
c_2	x_{12}	x_{22}	x_{32}	x_{42}	x_{m2}
c_3	x_{13}	x_{23}	x_{33}	x_{43}	x_{m3}
c_n	x_{1n}	x_{2n}	x_{3n}	x_{4n}	x_{mn}

Application procedure

- 6th Analyze the options attributes to verify if the lowest performance of some option, in fundamentally important criteria, make them unacceptable;
- 7th Determine or assign weights to the criteria;
- 8th Determine the criteria to be maximized and to be minimized and apply the exponential normalization procedure;

- Maximization: Higher values are the best condition

$$Exp_{ij} = \frac{e^{a \times x} - 1}{e^{a} - 1}, \ where \ x = \frac{x_{ij} - Min \ x_{ij}}{Max \ x_{ij} - Min \ x_{ij}}$$

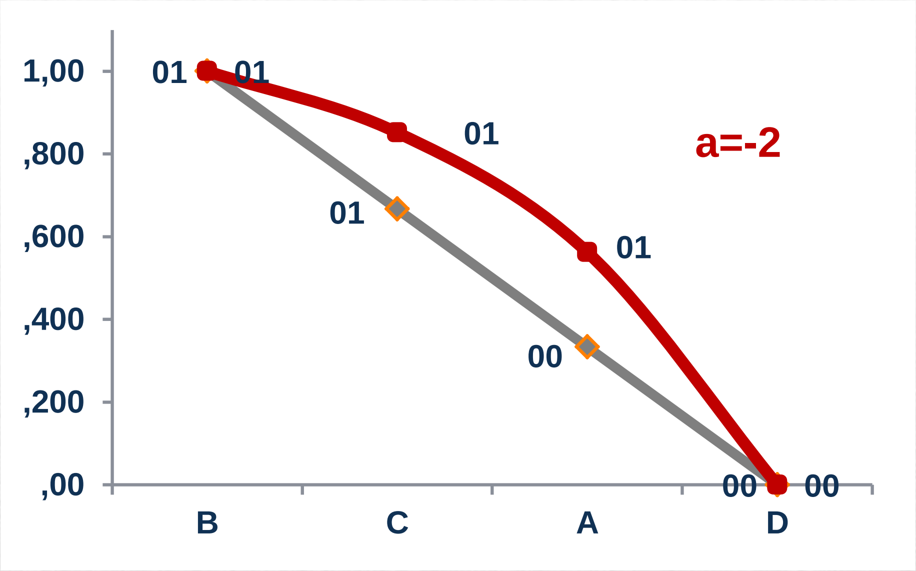
- Minimization: Lower values are the best condition

$$Exp_{ij} = \frac{e^{a \times x} - 1}{e^{a} - 1}, \ where \ x = \frac{Max \ x_{ij} - x_{ij}}{Max \ x_{ij} - Min \ x_{ij}}$$

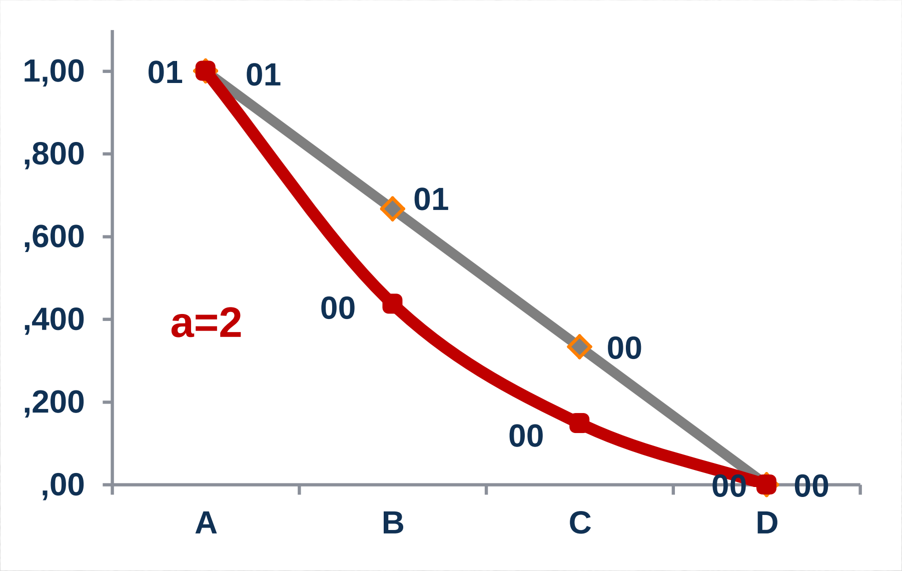
- x – corresponds to a linear normalization procedure
- a – corresponds to an independent factor

Application procedure

- A negative factor *a* results in a concave exponential growth

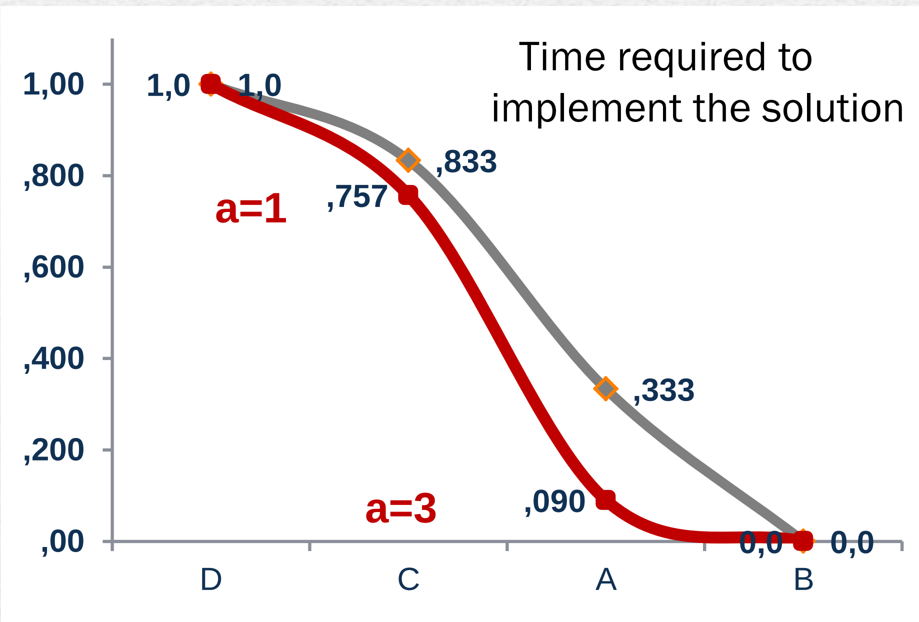


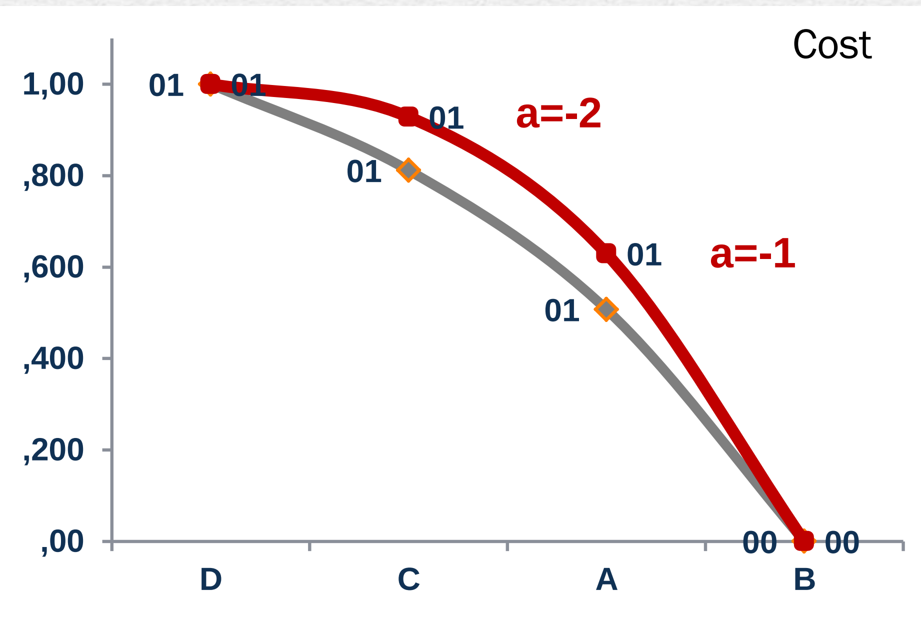
- A positive factor *a* results in a convex exponential growth



Application procedure

- 9th Analyze the lines progression and change factor *a* to reflect the decision-maker knowledge, preferences and objectives.



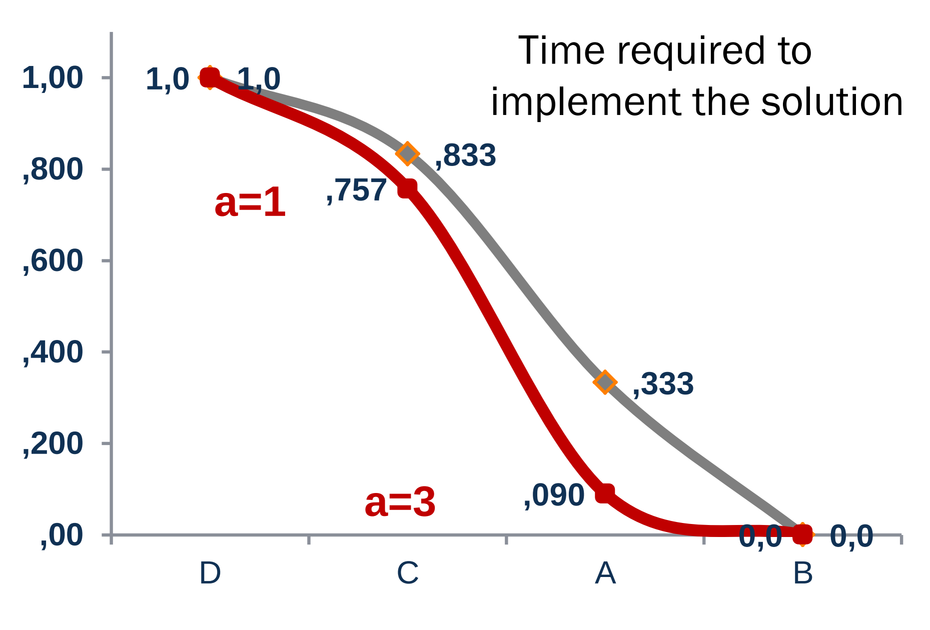


Application procedure

- The graph offers a good visual representation of the options relative value and we can make judgments having in mind all the options under evaluation.
- In this way, we are not only making paired-wise comparisons, we are also performing an integrated assessment of all the options since we can observe the relative position of all of them in the linear and on the exponential line.

Application procedure

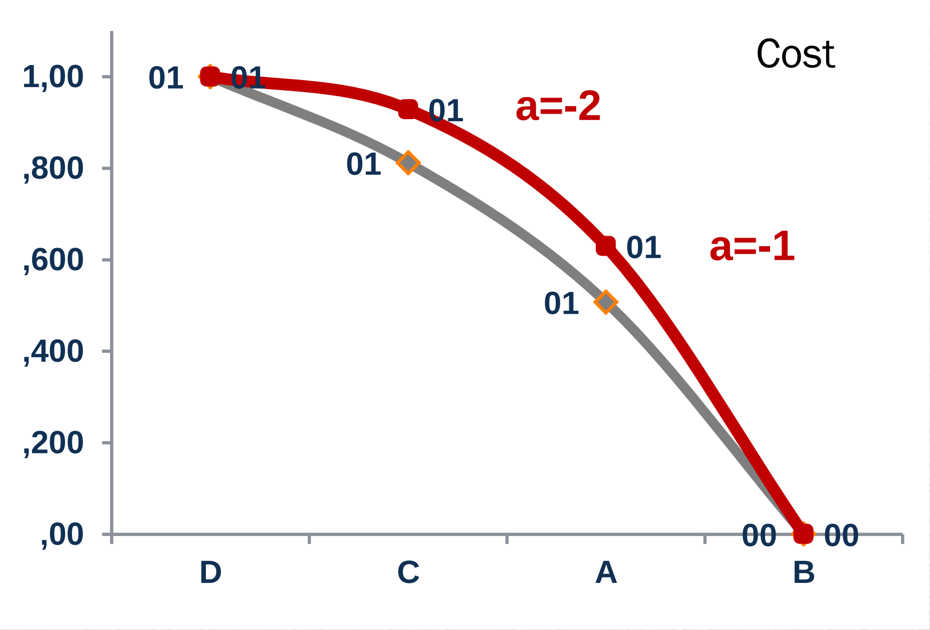
- 9th Analyze the lines progression and change factor *a* to reflect the decision-maker knowledge, preferences and objectives.



- Two factors *a* (1 and 3).
- Factor *a* equal to 1 expresses a decrease of value stronger than a linear evaluation would suggest;
- Factor *a* equal to 3 expresses a significant decrease of the value of option A, bringing it closer to the value of option B, which has the longest implementation time.

Application procedure

- 9th Analyze the lines progression and change factor *a* to reflect the decision-maker knowledge, preferences and objectives.



- Two factors *a* (-2 and -1).
- Factor *a* equal to -2 means that the decision-maker doesn't make a significant distinction between the cost of the first two options.
- As the cost increases, the decision-maker changes the negative factor *a* from -2 to -1 meaning that he still considers the option value a bit greater than the one assigned by a linear value line, detaching it from option B, which is the option with the highest cost.

Application procedure

Limitation of the normalization between 0 and 1:

- When we normalize the values between 0 and 1 the options inherent value is altered, for example:
  - Three students (A, B and C) have the following classifications: A(15), B(17) and C(18).
  - Since student A has the lowest classification (15) when we establish a normalization between 0 and 1, he will have 0 and student C with the best score (18) will have 1. If we multiply these values by the subject weight, student A has 0 on this curricular unit.
  - If we want to assess the course global performance of these three students, the student who pointed 0 will be penalized since the inherent value of his classification hasn't been taken into account.
- To recover the students classification intrinsic value we have to follow the procedure described in the 10^{th} step.

Application procedure

- 10th Determine the options relative value on every criterion: To determine the options relative value on every criterion we have to follow a four stages procedure:
  - 1st stage: Multiply the exponential normalization results by the difference between the criterion maximum and minimum value;
  - 2nd stage: Add the minimum criteria attribute to the previous results to re-establish the options inherent value;
  - 3rd stage: Establish the linear normalization for the attained options relative value;
  - 4th stage: Apply the previous process to all the remaining criteria to establish a normalized matrix.

Application procedure

- Numerical example:

Criteria attributes of every option									
	Condition	A	B	C	D	Sum	Min	Max	Max-Min
S1	Max	6	9	4	3	23	3	9	6
S2	Min	6	3	3	9	21	3	9	6
S3	Min	1	7	5	2	15	1	7	6
S4	Min	2	5	2	3	12	2	5	3

Exponential normalization					
	Factor *a*	A	B	C	D
S1	-1	0,622	1,000	0,243	0,000
S2	2	0,269	1,000	1,000	0,000
S3	2	1,000	0,000	0,148	0,672
S4	1	1,000	0,000	1,000	0,552

Application procedure

- Maximization procedure

Options	S1	Linear norm. (x)	*a*	Exp. norm. result	Max-Min	Exp. norm. result × (Max-Min)	Min	Options relative value	Linear norm.
B	9,00	1,000	-1	1,000	6	6,000	3	9,000	0,388
A	6,00	0,500	-1	0,622	6	3,735	3	6,735	0,290
C	4,00	0,167	-1	0,243	6	1,457	3	4,457	0,192
D	3,00	0,000	-1	0,000	6	0,000	3	3,000	0,129
								23,192	1

- Minimization procedure

Options	S2	Linear norm. (x)	*a*	Exp. norm. result	Max-Min	Exp. norm. result × (Max-Min)	Min	Options relative value	Linear norm.
B	3,00	1,00	2	1,000	6	6,000	3	9,000	0,351
A	3,00	1,00	2	1,000	6	6,000	3	9,000	0,351
C	6,00	0,50	2	0,269	6	1,614	3	4,614	0,180
D	9,00	0,00	2	0,000	6	0,000	3	3,000	0,117
								25,614	1

Application procedure

- 11^{th} Attain the options proportional value

	S1	S2	S3	S4
A	0,290	0,180	0,469	0,319
B	0,388	0,351	0,067	0,128
C	0,192	0,351	0,127	0,319
D	0,129	0,117	0,337	0,233

	Weight
S1	20
S2	35
S3	20
S4	25

	Options ranking	
A	29,48	1^{st}
B	24,59	3^{rd}
C	26,66	2^{nd}
D	19,27	4^{th}

$$
\begin{matrix} [Opt] & [S1] \quad [S2] \; ..... \; [Sn] & & [Crit\ W] & & [Ranking] \end{matrix}
$$

$$
\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}
\begin{bmatrix} A_{S1} & A_{S2} & ..... & A_{Sn} \\ B_{S1} & B_{S2} & ..... & B_{Sn} \\ C_{S1} & C_{S2} & ..... & C_{Sn} \\ D_{S1} & D_{S2} & ..... & D_{Sn} \end{bmatrix}
\times
\begin{bmatrix} W_{S1} \\ W_{S2} \\ ..... \\ W_{Sn} \end{bmatrix}
=
\begin{bmatrix} A_{S1} \times W_{S1} + A_{S2} \times W_{S2} + ... + A_{Sn} \times W_{Sn} \\ B_{S1} \times W_{S1} + B_{S2} \times W_{S2} + ... + B_{Sn} \times W_{Sn} \\ C_{S1} \times W_{S1} + C_{S2} \times W_{S2} + ... + C_{Sn} \times W_{Sn} \\ D_{S1} \times W_{S1} + D_{S2} \times W_{S2} + ... + D_{Sn} \times W_{Sn} \end{bmatrix}
$$

Application procedure

- **Nefarious values**
  - The PROV Exponential Decision Method allows the decision-maker to express his decrease of preference if at a determined level the continuous growth may become nefarious for the problem under analysis, such as very high or very low temperatures.
  - To determine the options relative value on every criterion when we have nefarious values we have to apply one of three procedures.

Application procedure

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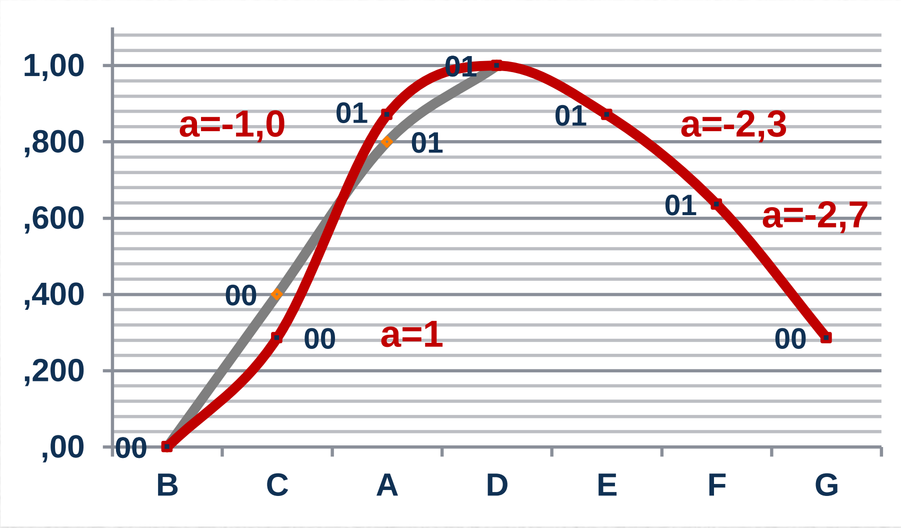
- Nefarious values
  - Maximize and minimize

This procedure is applied if we intend to maximize the criterion but at a determined value (D(14)), the preference starts decreasing. However its significance doesn't get as lower as the lowest value we are maximizing.

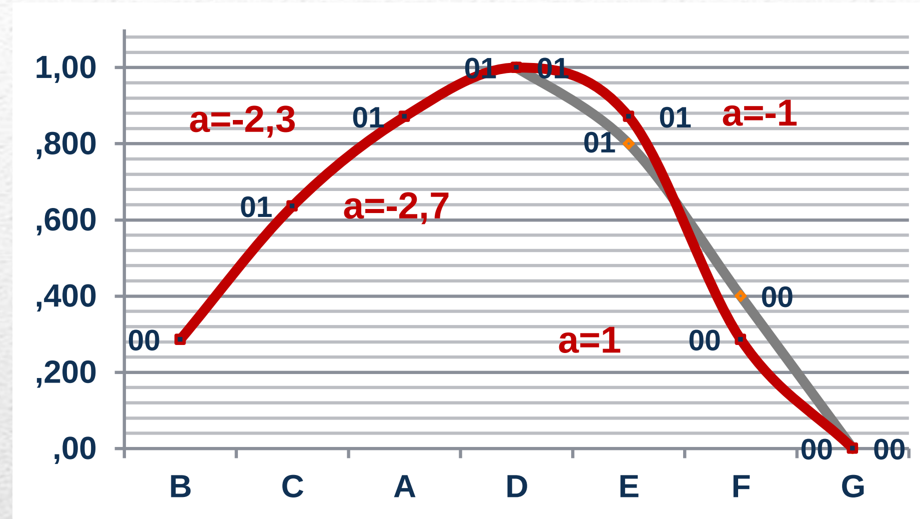


	Options	S1	Linear norm(x)	*a*	Exp. norm. result	Exp. norm. adjustment	Max-Min	Multp	Exp. Norm. × (Max-Min)	Options value	Linear Norm.
Max.	B	4	0,000	1,000	0,000	0,000	10	6	0,000	24,000	0,059
	C	8	0,400	1,000	0,286	0,286	10	6	17,174	41,174	0,102
	A	12	0,800	-1,000	0,871	0,871	10	6	52,269	76,269	0,188
	D	14	1,000	0,000	1,000	1,000	10	6	60,000	84,000	0,207
Min.	E	16	0,667	-2,292	0,871	0,871	6	10	52,264	76,264	0,188
	F	18	0,333	-2,700	0,636	0,636	6	10	38,171	62,171	0,153
	G	20	0,000	-2,700	0,000	0,286	6	10	17,174	41,174	0,102

Application procedure

- Nefarious values
  - Minimize and maximize

This procedure is applied if we intend to minimize the options criterion attributes to reach an optimal value (in this case option D (14)) but the continuous decrease bellow the optimal value starts to become nefarious.

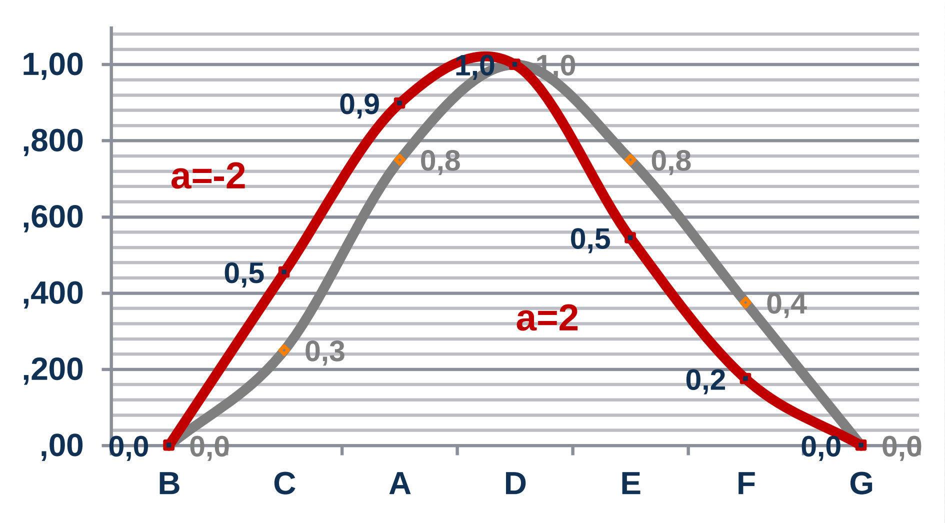


	Options	S1	Linear norm(x)	*a*	Exp. norm. result	Exp. norm. adjustment	Max-Min	Multp	Exp. Norm. × (Max-Min)	Options value	Linear Norm.
Min.	B	20	0,000	-2,700	0,000	0,286	6	10	0,000	24,000	0,062
	C	18	0,333	-2,700	0,636	0,636	6	10	38,171	62,171	0,160
	A	16	0,667	-2,292	0,871	0,871	6	10	52,264	76,264	0,197
	D	14	1,000	0,000	1,000	1,000	6	10	60,000	84,000	0,217
Max.	E	12	0,800	-1,000	0,871	0,871	10	6	52,269	76,269	0,197
	F	8	0,400	1,000	0,286	0,286	10	6	17,174	41,174	0,106
	G	4	0,000	1,000	0,000	0,000	10	6	0,000	24,000	0,062

Application procedure

- Nefarious values
  - Same minimum importance value

This procedure is applied if we intend to maximize the options criterion attributes but at a specific value (D(10)) the preference starts decreasing and its significance gets as lower as the lowest value we are maximizing.



	Options	S1	Linear norm(x)	*a*	Exp. norm. result	Max-Min	Multp	Denominator	Exp. Norm. × Denominator	Options value	Linear Norm.
Max.	B	2	0,000	-2	0,000	8	16	128	0,000	32,000	0,052
	C	4	0,250	-2	0,455	8	16	128	58,247	90,247	0,146
	A	8	0,750	-2	0,898	8	16	128	115,003	147,003	0,238
	D	10	1,000	0	1,000	8	16	128	128,000	160,000	0,259
Min.	E	14	0,750	2	0,545	16	8	128	69,753	101,753	0,165
	F	20	0,375	2	0,175	16	8	128	22,378	54,378	0,088
	G	26	0,000	2	0,000	16	8	128	0,000	32,000	0,052

Application procedure

The PROV Exponential Decision Method was developed to express the stakeholders knowledge, objectives and preferences to attain comprehensible results and to discover the most adequate solution for a problem or to accomplish a certain goal and the ordering and relative value of the alternatives of solution.

Take-home message

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Thanks for your attention!

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